GUIDED NOTES

Unit 7: Newtonian Mechanics – Oscillations

OBJECTIVES

Big Idea 3: The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.B: Classically, the acceleration of an object interacting with other objects can be predicted by using $\vec{a} = \frac{\vec{F}}{m}$.

Essential Knowledge 3.B.3: Restoring forces can result in oscillatory motion. When a linear restoring force is exerted on an object displaced from an equilibrium position, the object will undergo a special type of motion called simple harmonic motion. Examples should include gravitational force exerted by the Earth on a simple pendulum and mass-spring oscillator.

a. For a spring that exerts a linear restoring force, the period of a mass-spring oscillator increases with mass and decreases with spring stiffness.

b. For a simple pendulum, the period increases with the length of the pendulum and decreases with the magnitude of the gravitational field.

c. Minima, maxima, and zeros of position, velocity, and acceleration are features of harmonic motion. Students should be able to calculate force and acceleration for any given displacement for an object oscillating on a spring.

Learning Objective 3.B.3.1: The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties.

Learning Objective 3.B.3.2: The student is able to **design a plan and collect data** in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force.

Learning Objective 3.B.3.3: The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown.

Learning Objective 3.B.3.4: The student is able to construct a qualitative and/or a quantitative explanation of oscillatory behavior given evidence of a restoring force.

Enduring Understanding 3.C: At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.4: Contact forces result from the interaction of one object touching another object, and they arise from inter-atomic electrical forces. These forces include tension, friction, normal, spring, and buoyant.

GUIDED NOTES

NOTES:

I. ____________________________________

A. SHM is ___________________ motion, a repeated path back and forth around an equilibrium position.

- _________________________ position: The position at which an object in SHM _________________________________; often referred to as the "resting" or "relaxed" position

B. Two types of SHM:

1. _________________________ exhibit simple harmonic motion.

   a. An ideal spring ______________________________:

   …where ______ is the __________________ (sometimes called restoring force)

   b. Example 1: A mass attached to a spring (spring constant 1000 N/m) hangs at rest so that the spring is stretched 0.15 m. Some fun-loving person then pulls the mass downward 0.10 m from equilibrium. What's the spring force?

   c. Example 2: A mass-spring-Earth system hangs so that the mass and spring are at rest. Draw a free-body diagram to represent the forces acting on the mass. Write a one-sentence narrative describing the magnitudes and directions of these vectors.

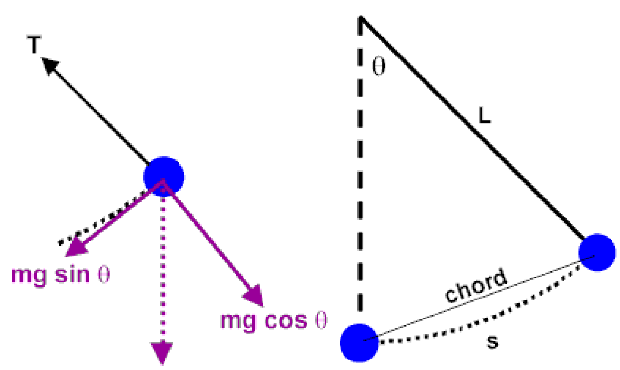
2. Pendulums essentially exhibit simple harmonic motion, although the interaction with gravity causes some variations in the motion.

GUIDED NOTES

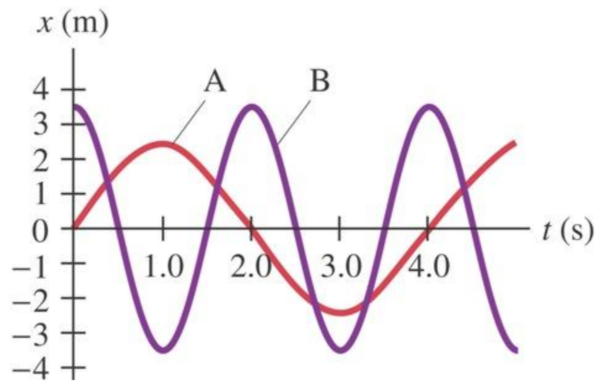
C. Describing simple harmonic motion

1. ________________________: Maximum displacement
    a. _____________________: The position of _________________________
    b. _____________________: The position of _________________________
2. _______________________: The displacement of the spring plotted against time creates a sine curve such that the displacement x at any time is found by
3. __________________of SHM
    a. Period for a _______________________:
    b. Period for a _______________________:**

** This is an approximation based on the idea that a pendulum pulled back at an angle has an arc that can be approximated using the sine of the angle, even though the swing is in a curve and not a straight line. The greater the angle, the less accurate this approximation is. (Calculus gives us a better method.) See http://dev.physicslab.org/asp/applets/javaphysmath/java/pend1/default.asp.



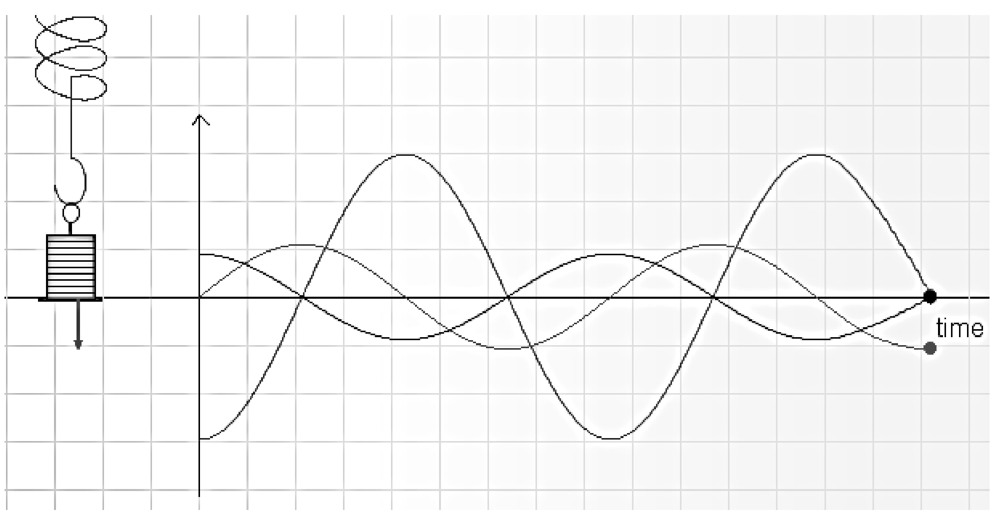
c. Example 3: Given the following graph of displacement as a function of time for a pendulum (a) determine the length of the pendulum A's cord and (b) determine the length of cord B in terms of the length of cord A.



Websites: PhET Wave on a String (set to "Oscillate", "No End", and set damping to zero.); UC Irvine

4. Reference circle[1]: We can use the motion of a rotating disk to explain SHM for a pendulum or mass-spring system. (Website: Online Physics Aplets)
   a. The radius of the circle corresponds to the amplitude.
   b. The angular (rotational) speed of the disc in radians/second is $\omega = 2\pi/T = 2\pi f$.
   c. The angle between the amplitude/displacement and the equilibrium line is θ.

MAKE A PREDICTION: Which line plotted below shows the displacement of a mass-spring system? Which line shows the velocity? Which line shows the acceleration?



[1] Not explicitly mentioned in the curriculum

5. Displacement, Velocity and Acceleration of SHM

   a. Displacement:

      1.) The displacement constantly changes.

      2.) At the maxima and minima, displacement is called amplitude, A.

   b. Velocity:

      1.) v_{max} when ____________________

      2.) v_{min} when _____________________

   c. Acceleration

      1.) a_{max} when _____________________

      2.) a_{min} when _____________________

   d. Website:

      1.) [Online Physics Aplets] (Show graph, not circle.)

      2.) [PhET Masses & Spring Lab]

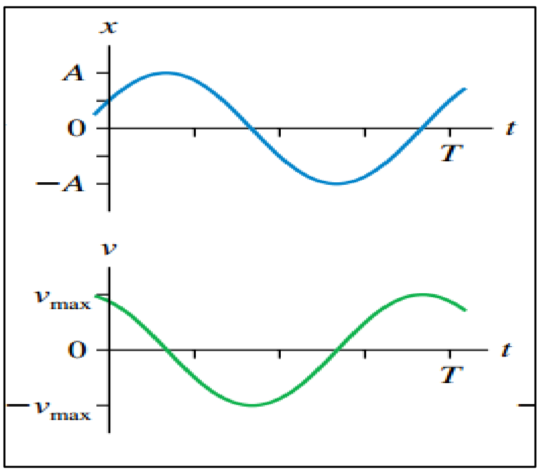
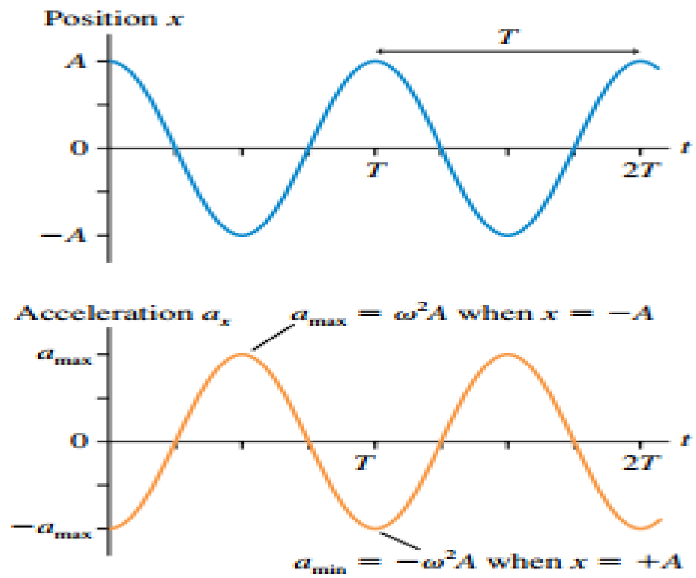
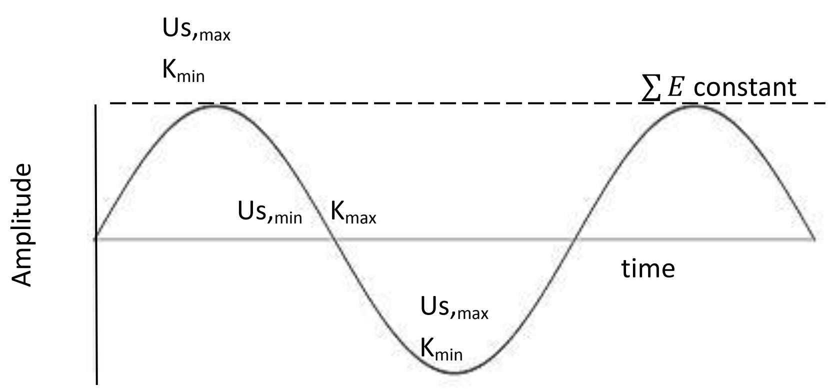
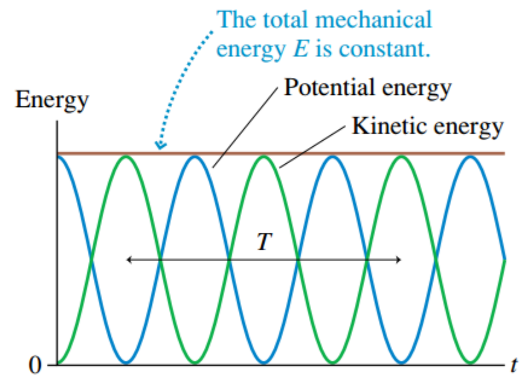


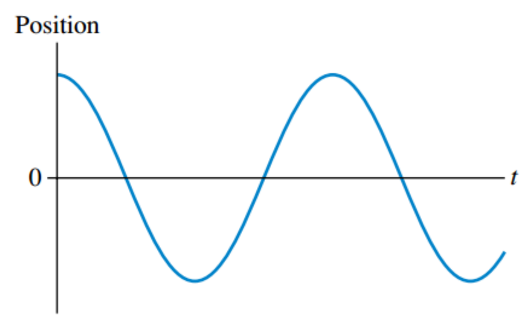
FIGURE 14.13 Position and acceleration graphs for an oscillating spring. We've chosen $\phi_0 = 0$.



6. Energy of an object oscillating in SHM
    a. Kinetic energy corresponds to ____________________.
    b. Elastic potential energy corresponds to ____________________________________.
    c. Gravitational potential energy corresponds to _______________________.
    d. ____.
    e. Key: ____________________________________ and _________________________ convert back and forth into each other.
    f. Analyzing graphs:







g. Example 4: A 0.100 kg mass-spring system with a spring of k = 600N/m is stretched with an amplitude of 0.044 m as the system oscillates horizontally on a friction-free surface once every 2.0 seconds. Find… (a) $U_{s,max}$ (b) total energy (c) work done to stretch the spring (d) K at equilibrium (e) velocity of spring as it passes equilibrium and (f) the maximum acceleration. (g) Sketch an accurate graph of amplitude and velocity as functions of time. (Only the dependent variables need to be accurate.) Finally (g) sketch an accurate graph of K, Us, and total energy. Align it with the top graph.

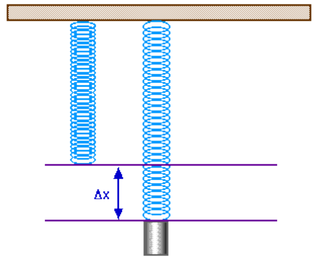
h. Vertical SHM is the same as horizontal SHM despite the role of gravity.

When a mass is hung vertically on a spring in a mass-spring-Earth system, the spring stretches such that the restoring force equals the weight of the mass. Hence we have

$$F_s = F_g$$

$$kx = mg$$

$$x = \frac{mg}{k}$$



If we then stretch the spring downward an additional distance x', we have a greater restoring force as follows.

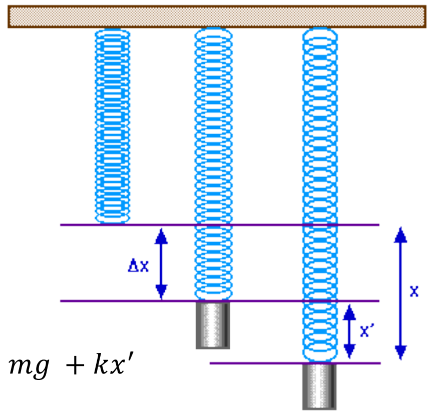
$$F_s = k(x + x')$$

We have already established that

$$x = \frac{mg}{k}$$

So

$$F_s = k(x + x') = kx + kx' = k\left(\frac{mg}{k}\right) + kx' = mg + kx'$$



In our free body diagram at this point, we have spring force upward and gravitational force downward, so the net force is

$$F_{net} = F_s - F_g = mg + kx' - mg$$

So in vertical SHM…

(taking the hanging length as the new equilibrium)

Moral of the story: ____ the ____________ of the vertical mass-spring-Earth system.

GUIDED NOTES

i. Example 5: The same mass-spring system is now hung vertically to become a mass-spring-Earth system. (a) How far is the spring stretched when the spring-mass system is at rest? (b) How much elastic potential energy exists in the system at this position? (c) Student A says there is zero elastic potential energy at this point. Explain how Student A can justify this. (d) The system is displaced 0.02 m downward from its resting length. Quantify the elastic potential energy from Student A's perspective. (e) As the system sets into oscillation, how can Student A find the maximum kinetic energy? Use a narrative to explain. (f) Solve for the maximum velocity of the system. (g) What is the maximum acceleration of the system? (h.) On the next page create four graphs with aligned x-axes. On one graph, plot displacement v time, labeling maxima and minima. On the next graph plot velocity v time. On the next graph plot acceleration v. time. On the next graph plot BOTH kinetic energy and total potential energy v time. Be sure to quantify the maxima and minima.

GUIDED NOTES

j. Damped & driven SHM[2]
   1.) When energy dissipates due to a damping force, such as friction, the SHM reduces due to lost energy. Example: shock absorber
   2.) Driven HM: When a force is applied to a mass in SHM at regular intervals in "resonance" with the frequency, the work being done by the force increases the total enrgy of the system, and the amplitude increases…unless there is also a damping force. Example: pushing a child on a swing

[2] Not explicitly mentioned in the curriculum