

GUIDED NOTES

Unit 8: Fluid Mechanics and Thermodynamics – Fluid Mechanics

OBJECTIVES:

Big Idea 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding 1.E: Materials have many macroscopic properties that result from the arrangement and interactions of the atoms and molecules that make up the material.

Essential Knowledge 1.E.1: Matter has a property called **density**.

Learning Objective 1.E.1.1: The student is able to predict the densities, difference in densities, or changes in densities under different conditions for natural phenomena and design an investigation to verify the prediction.

Learning Objective 1.E.1.2: The student is able to select from experimental data the information necessary to determine the density of an object and/or compare densities of several objects.

Big Idea 3: The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.C: At the macroscopic level, forces can be categorized as either long-range (action-at-a-distance) forces or contact forces.

Essential Knowledge 3.C.4: Contact forces result from the interaction of one object touching another object, and they arise from inter-atomic electric forces. These forces include tension, friction, normal, spring (Physics 1), and **buoyant** (Physics 2).

Learning Objective 3.C.4.1: The student is able to make claims about various contact forces between objects based on the microscopic cause of those forces.

Learning Objective 3.C.4.2: The student is able to explain contact forces (tension, friction, normal, **buoyant**, spring) as arising from inter-atomic electric forces and that they therefore have certain directions.

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.B: The energy of a system is conserved.

Essential Knowledge 5.B.10: **Bernoulli's equation** describes the conservation of energy in fluid flow.

Learning Objective 5.B.10.1: The student is able to use Bernoulli's equation to make calculations related to a moving fluid.

Learning Objective 5.B.10.2: The student is able to use Bernoulli's equation and/or the relationship between force and pressure to make calculations related to a moving fluid.

Learning Objective 5.B.10.3: The student is able to use Bernoulli's equation and the continuity equation to make calculations related to a moving fluid.

Learning Objective 5.B.10.4: The student is able to construct an explanation of Bernoulli's equation in terms of the conservation of energy.

Enduring Understanding 5.F: Classically, the mass of a system is conserved.

Essential Knowledge 5.F.1: The **continuity** equation describes conservation of mass flow rate in fluids. Examples should include **volume rate of flow** and **mass flow rate**.

Learning Objective 5.F.1.1: The student is able to make calculations of quantities related to flow of a fluid, using mass conservation principles (the continuity equation.)

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NOTES:

I. Hydrostatic Pressure

A. _____:

1. Symbol:

2. Formula:

- where m is _____ and
- V is _____.¹

3. SI Unit:

4. A _____

5. Density of water is _____ (not on the formula sheet)

B. _____: The _____

1. Symbol:

2. Formula:

- F is the magnitude of the _____ of the force exerted by the fluid on a surface
- A is the _____ being acted on.

3. SI Unit: _____

- _____ = 1 atmosphere (which is the air pressure at sea level) (This is on the formula sheet.)

4. A _____ (In the equation, force refers to the magnitude of the force.)

¹ Notice that there is a geometry section of your formula sheet with equations for areas and volumes.

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5. Important properties of fluids:

a. _____

b. _____

C. The _____ between _____ in a liquid

1. _____

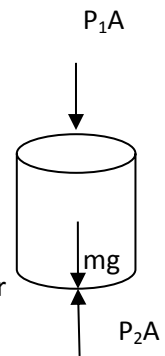
a. Consider a column of fluid in a container. Two forces act on it from outside, and one from inside.

1.) On the top: Pressure from air above $F_1 = P_1 A$

2.) From the support surface on the bottom: $F_2 = P_2 A$

3.) The weight of the column pushes down on the bottom surface:

$F_3 = mg$ where m is the mass of the water. The mass of the water equals density x volume: $m = \rho V = \rho Ah$ (based on the formula for the volume of a cylinder.)



$$\vec{F}_2 = \vec{F}_1 + mg$$

$$P_2 A = P_1 A + mg$$

$$P_2 A = P_1 A + \rho Ahg$$

$$P_2 = P_1 + \rho hg$$

Therefore, as we go deeper, pressure increases according to

$$\Delta P = \rho g \Delta h$$

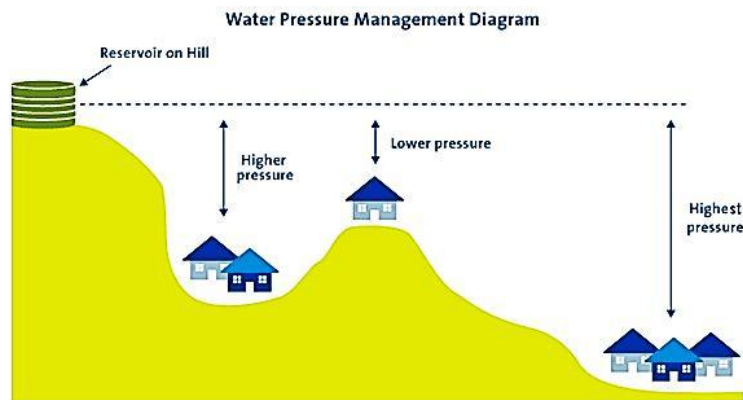
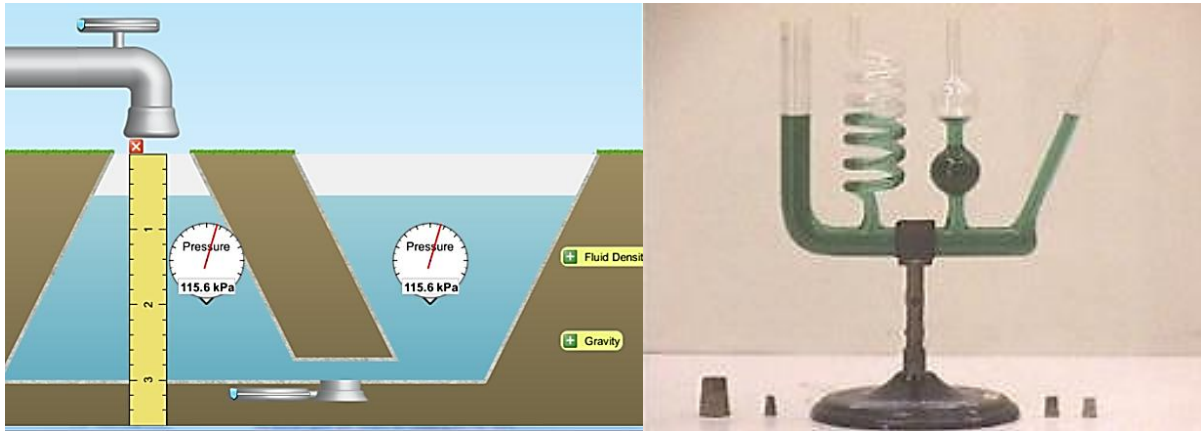
Such that as height changes...

(This is how it appears on your formula sheet.)

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b. The _____ in the water column _____.

<http://phet.colorado.edu/en/simulation/under-pressure>



c. Example 1: The surface of the water² in a storage tank is 30m above a water faucet in the kitchen of a house. How much greater is the pressure at the faucet than at the top of the tank?

² The density of water is 1000 kg/m^3 . This is not on the formula sheet.

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D. _____³

1. We can use pressure gauges with liquid inside of them to measure pressure.

2. A pressure gauge open to the atmosphere is

a _____.

3. A _____ uses pressure inside of the device

to push against the liquid in a _____ as follows...

a. _____: The pressure within the device (P_2 in our diagram)

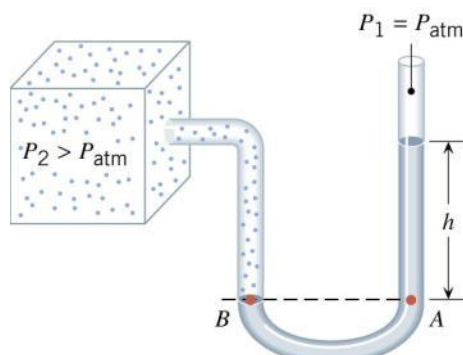
- For example, when taking blood pressure, the cuff is filled with air, creating the absolute pressure.

b. _____:

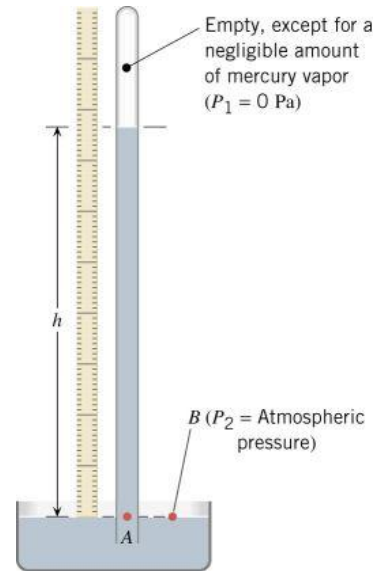
1.) The difference between P_2 and P_1 (i.e., ΔP , which equals $\rho g \Delta h$)

2.) This is how we measure pressure with a U-shaped tube. The reading from a pressure gauge gives the absolute (actual) pressure minus atmospheric pressure.

Why does it make sense to do it this way?



The pressures at points A & B are the same. ΔP is the gauge pressure, and it is what we measure.



³ This is not explicitly stated in the curriculum, but it is a reasonable application.

NOTICE THAT

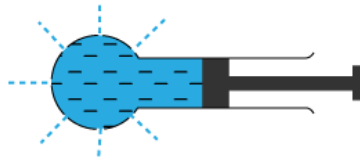
$$P_2 = P_1 + \rho gh$$

Absolute Pressure = Atmospheric Pressure + Gauge Pressure

E. _____⁴

1. This states that _____

is _____ to every part of the fluid, as well as to the walls of the container.

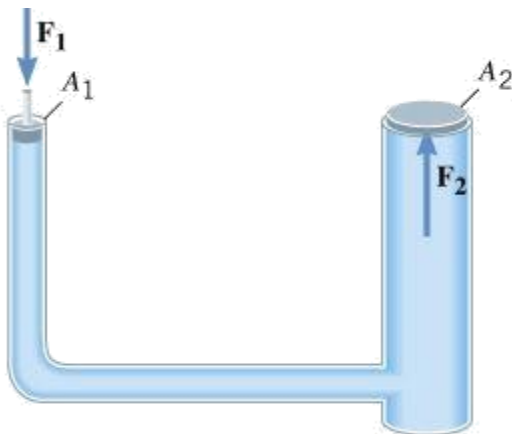


Pascal's Principle

In this image, water squirts out from all holes with equal force due to equal pressure.

2. Application: _____

The pressure on the right equals the pressure on the left. Therefore:



...thereby multiplying the force

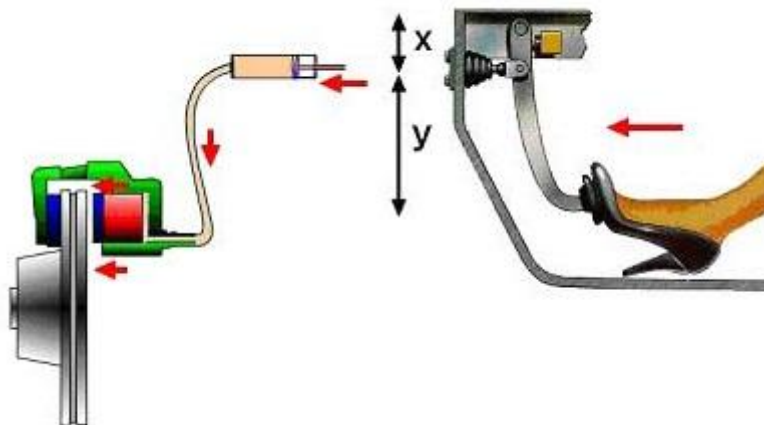
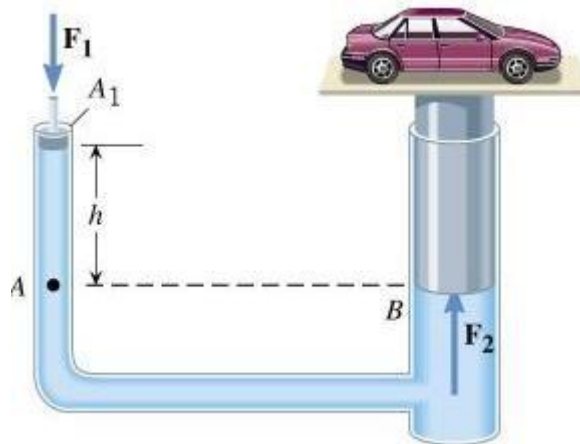
⁴ This is not explicitly stated in the curriculum, but it is a classic and essential fluid mechanics concept.

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For a car lift, the output plunger on the right is lower than the input piston on the left. Therefore there is even more pressure on the right piston because the pressure on the right piston equals

$$P_0 + \rho gh \text{ (where } P_0 \text{ is the pressure on the left piston.)}$$

We get a lot of “_____”⁵ using this method.



*This principle is how brake lines work in cars.
This diagram also shows why it is very hard to drive in heels.*

⁵ _____ is the _____.
It is a big deal, so highlight or circle this footnote. ☺

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3. Example 2: A hydraulic lift has one piston of diameter 0.1m and another piston of diameter 0.3m. (a) Which is the input piston? Why? (b) What force has to be applied to the input piston to lift a 1500kg car on top of the other piston? (c) When the small piston is displaced 5cm downward, how far is the car lifted?

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4. Example 3: Blaise Pascal (1623-1662) placed a long thin tube of $4\text{E-}3\text{m}$ radius vertically into a 0.2m radius barrel. He found that when the barrel was filled with water and the tube filled to a height of 10m , the barrel burst. Calculate (a) the pressure of the water at the level of the barrel lid and (b) the force the water applies upward on the barrel lid from within and (c) the net force on the lid of the barrel. (d) How does the volume of the liquid in the tube affect the result?⁶ (e) Why did the container not burst when it was filled with air?

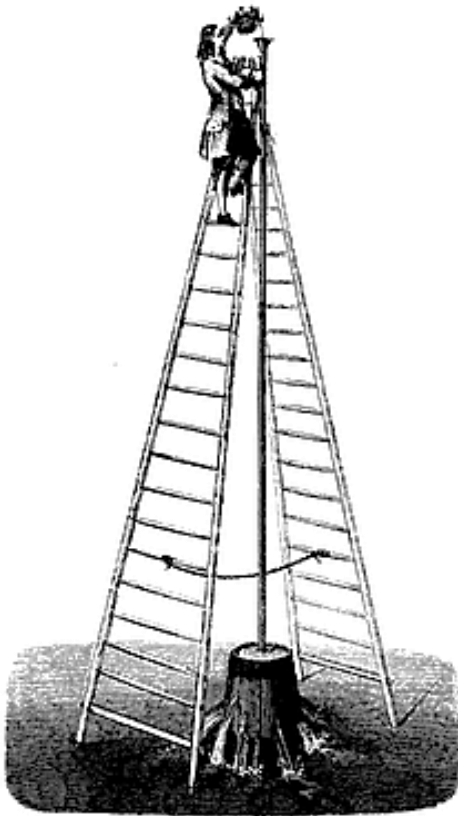


FIG. 43.—Hydrostatic paradox. Pascal's experiment.

⁶ Remember that $P = P_0 + \rho gh$, so pressure does not depend on volume. The tube had a huge amount of pressure inside of it, AND pressure is equally distributed throughout the liquid.)

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II. Buoyancy

A. _____: The upward force on a submerged object

B. _____:

1. The _____



This is because the upward force on an object (being deeper) is greater than the downward force. Therefore the buoyant force is the bottom force minus the top force, making the following equations true...

$$F_B = P_2A - P_1A$$

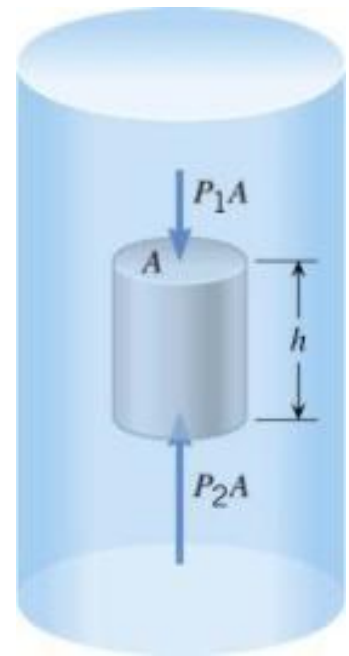
$$F_B = (P_2 - P_1)A$$

$$F_B = \Delta PA$$

$$F_B = \rho ghA$$

$$F_B = \rho gV$$

which is the same as



(which is how it appears on your formula sheet)

...and REMEMBER also...

of the displaced fluid (NOT of mg of the object.)

(That is not on the formula sheet but is derived from the density formula where $\rho = \frac{m}{V} \rightarrow m = \rho V$)

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3. _____: Rules...

a. The _____ of the object

_____.

b. Flotation depends on whether the buoyant force is $>$, $<$, or $=$ to the weight of the object.

Flotation Fail: Object sinks

The object is

The _____ of the object is
_____ the mass of
the displaced water.

The _____ of the object
is _____ the weight
of the displaced water:

Flotation 1: Object submerges fully & hovers in the water

The object has the

The _____ of the object is
_____ the mass of the displaced
water.

The _____ of the object
is _____ the weight
of the displaced water:

Flotation 2: Object is partially submerged.

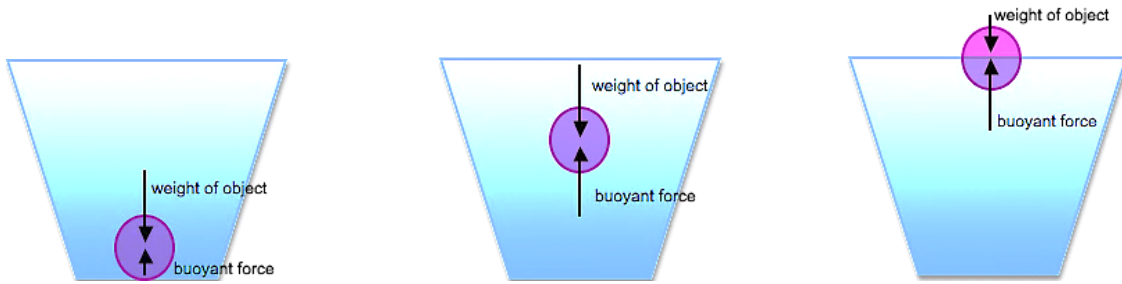
The object is

The _____ of the object is
still
_____ the mass of the displaced
water. It's just that the
is _____.

The _____ of the object
is _____ the weight
of an _____ of
water but still equals the
buoyant force:

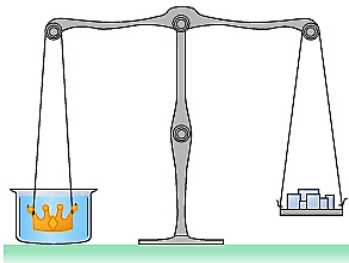
**The F_B equals the weight of
the displaced volume of
water. This volume of water
equals the volume of only
the submerged portion of
the object.**

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4. Example 4: A 70kg ancient statue from the lost city of Atlantis lies at the bottom of the sea. Its volume is $3.0\text{E}-2\text{m}^3$. (a) Does it hover, partially submerge, or sink? The density of sea water is $1.025\text{E}3\text{kg/m}^3$. (b) A boat attaches a cable to the statue in order to lift it. How much tension is in the cable as the statue moves upward at a constant velocity? (Be sure to show a free body diagram in your work.)

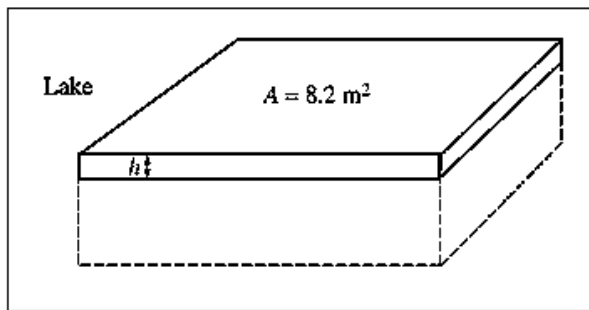
5. Example 5: When a crown of mass 14.7kg is submerged in water, an accurate scale reads only 13.4kg. (a) Draw a free body diagram for the forces acting on the crown. (b) What is the mass of the displaced water? (c) What is the volume of the displaced water? (d) What is the volume of the crown? (e) Is the crown made of gold? The density of pure gold is $1.93\text{E}4\text{kg/m}^3$.



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6. Example 6: A large rectangular raft (density 650 kg/m^3) is floating on a lake. The surface area of the top of the raft is 8.2 m^2 and its volume is 1.80 m^3 . The density of the lake water is 1000 kg/m^3 .⁷

- Calculate the height of the portion of the raft that is above the surrounding water.
- Calculate the magnitude of the buoyant force on the raft and state its direction.
- If the average mass of a person is 75 kg , calculate the maximum number of people that can be on the raft without the top of the raft sinking below the surface of the water. (Assume that the people are evenly distributed on the raft.)



Note: Figure not drawn to scale.

⁷ This is a 2005 FRQ.

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III. _____

A. Concepts

1. _____: The velocity of a fluid passing a specific point is constant.
2. _____: A fluid whose _____
when pressure changes.⁸
3. _____: This refers to the _____ of a fluid. A fluid with high viscosity has a lot of _____ causing it not to flow well.
4. _____: A fluid that _____ with
_____ due to W_{nc} from collisions between the fluid's particles

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.F: Classically, the mass of a system is conserved.

Essential Knowledge 5.F.1: The continuity equation describes conservation of mass flow rate in fluids. Examples should include volume rate of flow and mass flow rate.

B. _____: the velocity of a fluid expressed in terms of mass passing a point in a second

1. Mass flow rate =

(This is not on your formula sheet, but if you know it as a concept, you will be OK.)

2. SI Unit:

⁸ Most of the time we will simplify matters by considering fluids to be incompressible even though no fluid is really incompressible. However, not to consider fluids to be incompressible at this stage in our learning would make the problems about non-incompressible fluids incomprehensible. ;^D

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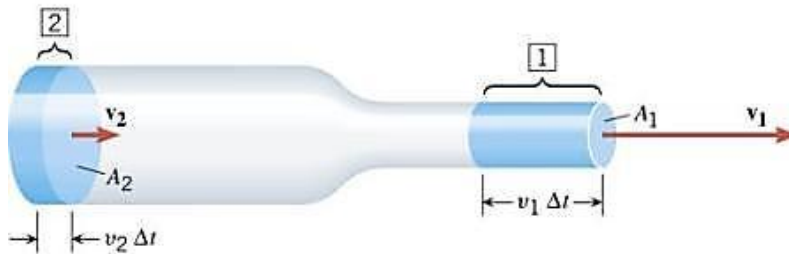
C. _____: the volume flow rate, the volume of fluid per second passing through the tube.

1.

(This is also not on your formula sheet, but if you know it as a concept, you will be OK.)

2. SI Unit:

D. _____: The mass _____ (or the volume flow rate) has the _____ at _____ along a tube that has a single entry and a single exit point for fluid flow.



- A fancy derivation: $\frac{\Delta m}{\Delta t} = \frac{\rho V}{\Delta t} = \frac{\rho A h}{\Delta t} = \rho A \frac{h}{\Delta t} = \rho A v$
(Note that the units on both sides of the equation are kg/s.)
- At point 1: $\frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1$
- At point 2: $\frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2$

- Equation of Continuity:

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Now...if the fluid is incompressible ρ cancels, and life is butterflies and sunshine.

We then get...

$$A_1 v_1 = A_2 v_2$$

which is on your formula sheet, and it amounts to

$$Q_1 = Q_2$$

which is not on your formula sheet, since it's just the concept.

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- Application: Cross-sectional area and velocity are inversely proportional. Fluids flow faster through thinner pathways. This is PRETTY IMPORTANT SO I'M GOING ALL CAPS.

E. Examples:

1. Example 6: The radius of the aorta is about $1.0 \times 10^{-2} \text{ m}$ and the blood passing through it has a speed of about 0.30 m/s . A typical capillary has a radius of about $4 \times 10^{-6} \text{ m}$, and blood flows through it at a speed of about $5 \times 10^{-4} \text{ m/s}$. (a) Construct a sketch to represent how this pertains to fluid flow continuity. (b) Estimate how many capillaries there are in the body.

2. Example 7: Heating ducts are the tubes that push air into rooms. (a) How large (cross-sectional area) must a heating duct be if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of 300 m^3 volume? Assume the air is incompressible. (b) Where does the air flow the fastest? Relate this to your experience of heating vents.

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Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

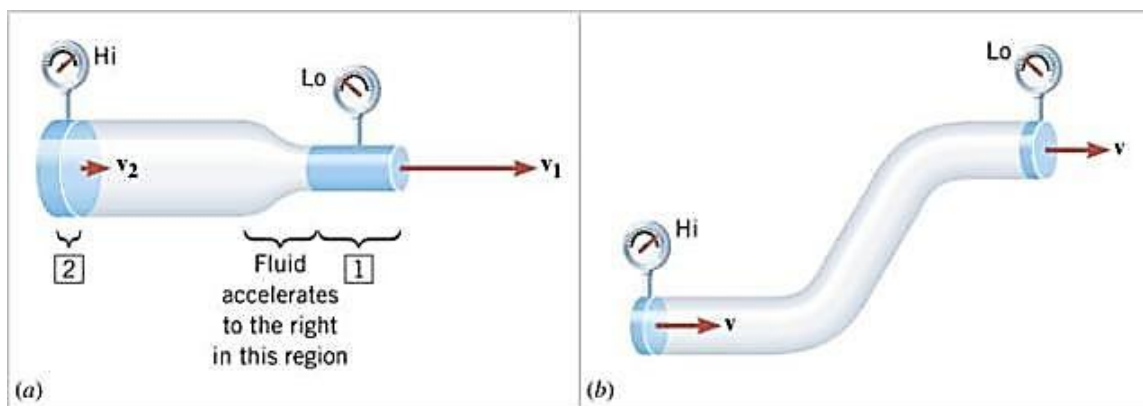
Enduring Understanding 5.B: The energy of a system is conserved.

Essential Knowledge 5.B.10: Bernoulli's equation describes the conservation of energy in fluid flow.

IV. _____: Daniel Bernoulli (1700-1782) discovered an equation that takes fluid flow continuity one step further to look at changing elevations. This "Bernoulli's equation" relates the speed, pressure, and elevation of an incompressible, nonviscous fluid in steady flow.

A. Concept

- Since $A_1v_1 = A_2v_2$ the velocity in a thin portion of a tube is large. Therefore fluid entering a bottleneck must accelerate.
- By Newton's 2nd Law, there must therefore be a greater force in the larger portion of the tube.
- Hence, pressure is higher in a region of larger cross-sectional area.
- In addition, if the fluid moves to a higher elevation, the pressure at the lower level is greater than the pressure at the higher level as has been shown prior.
- Together all of these comprise Bernoulli's equation.



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B. Bernoulli's Equation:

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Note: When a pipe is horizontal so that $y_1 = y_2$, then the equation is simply

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

(but only if the pipe is horizontal, so don't go getting confused and using

this when the pipe has vertical lift to it.)

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C. Bernoulli's equation as conservation of energy

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

$$P_1 + U_{g,o} + K_0 = P_2 + U_g + K$$

where

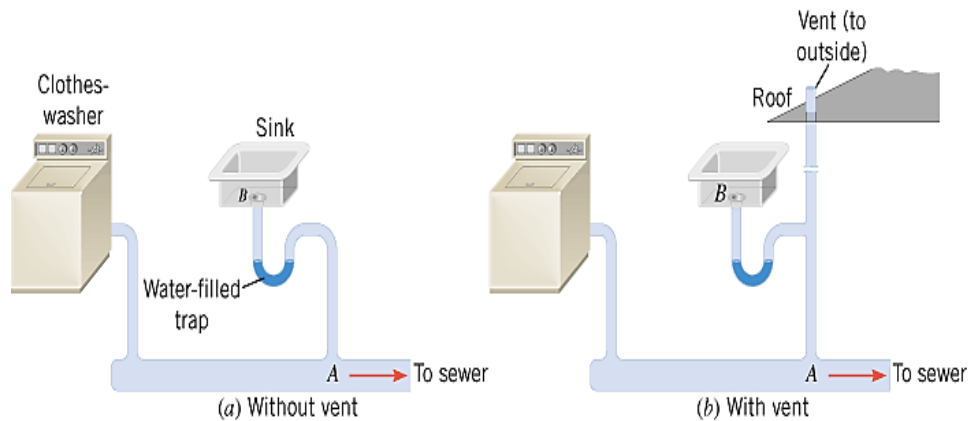
- U_g can be considered to be gravitational potential energy *per unit of volume*, and K can similarly be considered to be kinetic energy *per unit of volume*.
- Pressure in this sense can be considered to be energy density, or energy per unit volume.

$$(\text{Derivation: } P = \frac{F}{A} = \frac{Fx}{Ax} = \frac{\text{Work}}{\text{Volume}} = \frac{\text{Energy}}{\text{Volume}})$$

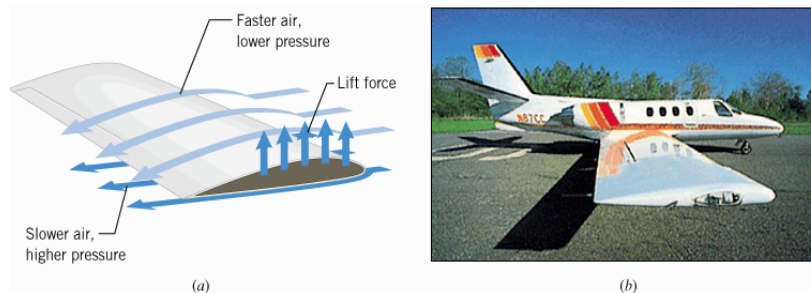
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C. Applications:

1. Household plumbing: Ever wonder why there are U-shaped pipes under sinks and toilets? The water “plug” prevents sewer gas from entering the house, which is a pretty good thing. Analyze the picture below to explain in terms of Bernoulli’s equation why the vented arrangement shown in (b) prevents to water plug from draining when the clothes washer drains.



2. Airplane wing: The curved top makes air move faster over it, creating low pressure and lift.



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D. Examples

1. Example 8: Water circulates throughout a house in a hot-water heating system. The water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm . (a) Create a sketch to show this scenario and write in numbers where appropriate. (b) Should the flow speed be higher, lower, or the same in a pipe with a diameter of 2.6 cm on the second floor 5.0 m above the pipe in the basement? Justify your answer semiquantitatively. (c) Now quantify your answer. (d) Should the pressure be higher, lower, or the same in the upper pipe compared with the lower pipe? Justify your answer. (e) Now quantify your answer.

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2. Example 9: For homework, write about how the principles related to fluid mechanics affect the design of the water towers that supply municipalities with water. Consider what the goals should be in terms of delivering water to citizens. Background: At point B we have a pump that receives water from the water treatment plant (point A) and pushes water out to homes, industry, and business (point C.) For most of the day, the pump provides enough pressure to get plenty of water to where it's needed. At "peak usage" times, such as in the mornings, cities would need a pretty big pump to get water everywhere it's needed. However, pumps are expensive, so cities don't want to put in costly stronger pumps just to meet the needs of peak times when, for the rest of the day, that kind of pumping power isn't needed. Enter the water tower. At low usage times, the pump pushes the water level up at point D because not as much water is flowing to homes and businesses (point C.) At peak times, when the pump can't push enough water to point C to meet demand, the water tower provides extra water to the city. So...use the principles we've learned about in this unit to write an AP Physics 2 exam-quality answer to the question: How do the principles related to fluid mechanics affect the design of water towers? (We will score this with a specific rubric, so make this awesome.)

