GUIDED NOTES

Unit 5: Newtonian Mechanics – Linear Momentum

OBJECTIVES

We will continue to reinforce the all concepts explored previously.

Big Idea 3: The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.D: A force exerted on an object can change the momentum of the object.

Essential Knowledge 3.D.1: The change in momentum of an object is a vector in the direction of the net force exerted on the object.

Learning Objective 3.D.1.1: The student is able to justify the selection of data needed to determine the relationship between the direction of the force acting on an object and the change in momentum caused by that force.

Essential Knowledge 3.D.2: The change in momentum of an object occurs over a time interval.

a. The force that one object exerts on a second object changes the momentum of the second object (in the absence of other forces on the second object.)

b. The change in momentum of that object depends on the impulse, which is the product of the average force and the time interval during which the interaction occurred.

Learning Objective 3.D.2.1: The student is able to justify the selection of routines for the calculation of the relationships between changes in momentum of an object, average force, impulse, and time of interaction.

Learning Objective 3.D.2.2: The student is able to predict the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted.

Learning Objective 3.D.2.3: The student is able to analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted.

Learning Objective 3.D.2.4: The student is able to **design a plan** for collecting data to investigate the relationship between changes in momentum and the average force exerted on an object over time.

Enduring Understanding 3.E: A force exerted on an object can change the kinetic energy of the object.

* Everything we did in our energy unit related to this Enduring Understanding will be revisited in this unit.

Big Idea 4: Interactions between systems can result in changes in those systems.

Enduring Understanding 4.A: The acceleration of the center of mass of a system is related to the net force exerted on the system, where $\vec{a} = \frac{\vec{F}}{m}$.

Essential Knowledge 4.A.1: The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass.

Learning Objective 4.A.1.1: The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semiquantitatively.

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Enduring Understanding 4.B: Interactions with other objects or systems can change the total linear momentum of a system.

Essential Knowledge 4.B.1: The change in linear momentum for a constant-mass system is the product of the mass of the system and the change in velocity of the center of mass.

Learning Objective 4.B.1.1: The student is able to calculate the change in linear momentum of a two-object system with constant mass in linear motion from a representation of the system (data, graphs, etc.)

Learning Objective 4.B.1.2: The student is able to analyze data to find the change in linear momentum for a constant-mass system using the product of the mass and the change in velocity of the center of mass.

Essential Knowledge 4.B.2: The change in linear momentum of the system is given by the product of the average force on that system and the time interval during which the force is exerted.

a. The units for momentum are the same as the units of the areas under the curve of a force versus time graph.

b. The changes in linear momentum and force are both vectors in the same direction.

Learning Objective 4.B.2.1: The student is able to apply mathematical routines to calculate the change in momentum of a system by analyzing the average force exerted over a certain time on the system.

Learning Objective 4.B.2.2: The student is able to perform analysis on data presented as a force-time graph and predict the change in momentum of a system.

Enduring Understanding 4.C: Interactions with other objects or systems can change the total energy of a system.

* Everything we did in our energy unit related to this Enduring Understanding will be revisited in this unit.

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A: Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.1: A system is an object or a collection of objects. The objects are treated as having no internal structure.

Essential Knowledge 5.A.2: For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserve quantity with its surroundings.

Learning Objective 5.A.2.1: The student is able to define open and closed/isolated systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

Essential Knowledge 5.A.3: An interaction can be either a force exerted by objects outside the system or the transfer of some quantity with objects outside the system.

Essential Knowledge 5.A.4: The boundary between a system and its environment is a decision made by the person considering the situation in order to simplify or otherwise assist in analysis.

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Enduring Understanding 5.D: The linear momentum of a system is conserved.

Essential Knowledge 5.D.1: In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after.

a. In an isolated system, the linear momentum is constant throughout the collision.

b. In an isolated system, the kinetic energy after an elastic collision is the same as the kinetic energy before the collision.

Learning Objective 5.D.1.1: The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions.

Learning Objective 5.D.1.2: The student is able to apply the principles of conservation of momentum and restoration of kinetic energy to reconcile a situation that appears to be isolated and elastic, but in which data indicate that linear momentum and kinetic energy are not the same after the interaction, by refining a scientific question to identify interactions that have not been considered. Students will be expected to solve qualitatively and/or quantitatively for one-dimensional situations and only qualitatively in two-dimensional situations.

Learning Objective 5.D.1.3: The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy.

Learning Objective 5.D.1.4: The student is able to design an experimental test of an application of the principle of the conservation of linear momentum, predict an outcome of the experiment using the principle, analyze data generated by that experiment whose uncertainties are expressed numerically, and evaluate the match between the prediction and the outcome.

Learning Objective 5.D.1.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values.

Learning Objective 5.D.1.6: The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed.

Learning Objective 5.D.1.7: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values.

Essential Knowledge 5.D.2: In a collision between objects, linear momentum is conserved. In an inelastic collision, kinetic energy is not the same before and after the collision.

a. In an isolated system, the linear momentum is constant throughout the collision.

b. In an isolated system, the kinetic energy after an inelastic collision is different from the kinetic energy before the collision.

Learning Objective 5.D.2.1: The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic.

Learning Objective 5.D.2.2: The student is able to plan data collection strategies to test the law of conservation of momentum in a two-object collision that is elastic or inelastic and analyze the resulting data graphically.

Learning Objective 5.D.2.3: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy.

Learning Objective 5.D.2.4: The student is able to analyze data that verify conservation of momentum in collisions with and without an external friction force.

Learning Objective 5.D.2.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values.

Learning Objective 5.D.2.6: The student is able to apply the conservation of linear momentum to an isolated system of objects involved in an inelastic collision to predict the change in kinetic energy.

Essential Knowledge 5.D.3: The velocity of the center of mass of the system cannot be changed by an interaction within the system.

a. The center of mass of a system depends upon the masses and positions of the objects in the system. In an isolated system (a system with no external forces), the velocity of the center of mass does not change.

b. When objects in a system collide, the velocity of the center of mass of the system will not change unless an external force is exerted on the system.

Learning Objective 5.D.3.1: The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force).

Learning Objective 5.D.3.2: The student is able to make predictions about the velocity of the center of mass for interactions within a defined one-dimensional system.

Learning Objective 5.D.3.3: The student is able to make predictions about the velocity of the center of mass for interactions within a defined two-dimensional system.

GUIDED NOTES

NOTES:

I. Impulse-Momentum Theorem

A. __________________________: The _________ of an _________ to __________________ its straight-line _________.

1. Symbol:
2. Formula:

3. SI Unit:
4. A _________
5. Example 1: A 0.145kg baseball moves at 20m/s. What's its momentum?

B. __________________: An impulse occurs when an _________________________ acts on an ____________ and _________________________ in the object's ____________. An impulse is the product of the average external force and the time interval during which the interaction occurred.

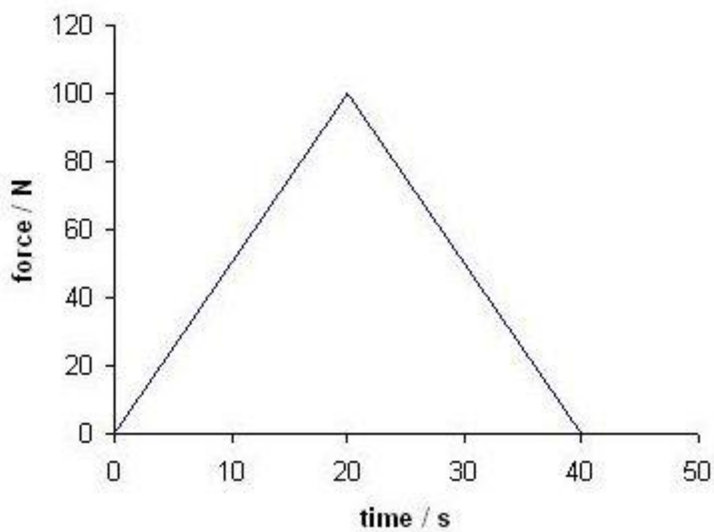
1. Symbol: (not used on the formula sheet)
2. Formula:

(The formula sheet leaves off the $\vec{J}$.)

3. SI Unit: _________________________which is ____________ (Keep this in mind!)
4. A ____________ in the direction of the net force exerted on the object.

5. <u>Example 2</u>: A tennis racket applies 80N of force to a 0.058 kg tennis ball in 0.1 seconds. (a) What impulse did the racket apply to the ball? (b) What was the system affected by the impulse?(c) Sketch a vector for impulse next to a free-body diagram to represent the situation.

6. The ________________________of a _________________________equals ____________.



<u>Example 3</u>: Calculate the impulse exerted on an object as shown in the above graph. Then generate a narrative problem that could go along with this, including a description of the system and an identification of the source of the external force. Finally, relate the units from this graph to the units of momentum.

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<u>Big Idea 4</u>: Interactions between systems can result in changes in those systems.

C. The ____________________________________ states that ____.

1. A force exerted on an object by another object can change the momentum of the object.[1] This change occurs over the time interval during which the force was exerted.
   - Question: What is the system?
   - Question: Where does the force come from?
   - Question: How does our view change if we define the system differently?

2. Expanding on the formula:

3. (Again, only the center part is on the formula sheet.)

   <u>Note: This assumes the system to be a constant-mass system.</u>

4. Generate the derivation of this theorem using previously-learned relationships. (F=ma…)

[1] In the absence of any other external forces acting on the second object.

5. Example 4: Return to example 3 and add to your narrative a description of the change in linear momentum of the object.

6. Example 5: A 70kg man doing Thor cosplay jumps upward with a velocity of 1.5 m/s from a height of 3.0m and lands on firm ground. (a) How fast is he going when he lands? (b) What impulse does the ground apply to the pseudo-Thor to stop him? (c) If he lands stiff-legged, the body moves 1.0cm during the impact. How much time does it take him to stop? (d) Therefore how much force is required to stop him? (e) Draw a free-body diagram to represent pseudo-Thor's landing. Next to the diagram draw impulse, initial momentum, acceleration, and initial velocity vectors that also correspond to his landing. (f) If instead he bends his knees during the collision with the ground and therefore comes to a top over 50cm, what force does the ground apply to him?

7. Alterations in force and time
   a. ___________________ momentum safely: Change momentum over an _________________________to ____________ the required ____________:

   b. ______________________________in momentum: Apply a ____________________ for an extended time (e.g., ____________________ when hitting a baseball) to create a big impulse.
8. What about an external force acting on a system with internal structure?
   Example 6: A two-car train moves along a straight track with a velocity of 1.5 m/s. Consider both cars to be rigid and symmetrical. Also consider the connecting link between the cars to be rigid. Car A's mass is 2000 kg, and Car B's mass is 4000 kg. Their center-to-center distance is 15 m. (a) Where is their *cm*? [2] (b) What is the momentum of each car? Of the system? (c) What impulse is required to stop the system? (d) The train collides with a rigid bumper in a roundhouse. What force is required to stop the system in 1.9 seconds? (e) Generate at least five *accurate* graphs to represent this situation.

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[2] $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 4.B: Interactions with other objects or systems can change the total linear momentum of a system.

II. ______________________________________

A. ____________________________, the ____________________________ of the __________ remains ___________________ as long as ____________________________ on the system (i.e., the system is ____________.) Momentum is ______________________.

1. When the motion is one-dimensional, momentum is conserved in that dimension. (See examples 7 and 8, below.)
2. When the motion is two-dimensional, momentum is conserved in *each* dimension. (See example 9, below.)[3]

B. Generate a formula that would express this. Go shallow enough to show the general relationship and deep enough to include initial and final velocities.

C. Examples

1. Example 7: A 10,000kg train car traveling at a speed of 24.0m/s strikes an identical car at rest. If the cars lock together as a result of the collision, what is their common speed afterward?

[3] http://hyperphysics.phy-astr.gsu.edu/hbase/inecol4.html: You can set up collisions and predict the values for practice.

2. Example 8: Calculate the recoil velocity of a 5.0kg rifle that shoots a 0.050kg bullet at a speed of 120m/s.

3. Example 9: (a) Before trying the math, generate a problem-solving strategy for a two-dimensional collision problem. (b) A particle traveling at 10m/s at an angle of 30° above the horizontal collides with a particle moving upward at an angle of 20° to the horizontal with a velocity of 12m/s. Afterwards the first particle moves at an angle of 40° above horizontal with a velocity of 10m/s. Both particles have the same mass. What is the final velocity of the second particle?

GUIDED NOTES

III. Energy and collisions

A. ____________________

1. In an ____________________, the ______________________________r an elastic collision ____________________ as the kinetic energy __________ the collision.

    1. What __________ must be met in order for kinetic energy to be conserved?

        a. There must be ____, such as an external friction force.

        b. There must be ___________transferring energy from the system to the environment.

        c. ______________________________, there must be ___________________ between the objects and ____________________on the objects (____________________.) Therefore objects cannot stick together and must ____________________________ after the collision.

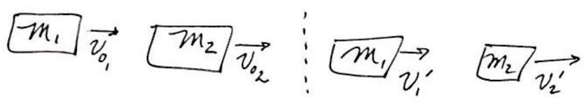
    2. <u>Example 10</u>: A 2kg mass moving east at 10m/s collides with a westward-moving 7kg mass traveling at 4m/s in an isolated system. The 7kg block moves eastward at 2.2m/s after the collision. Is this collision elastic?

<u>BIG RULE ALERT</u> When a final velocity is unknown in a collision problem, always use ____ to find the unknown velocity.

3. What if you don't know either final velocities?

   Shortcut: The approach velocity equals the departure velocity by

   by…



momentum:

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1' + m_2 v_2'$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_{02})$$

KE:

$$\tfrac{1}{2} m_1 v_{01}^2 + \tfrac{1}{2} m_2 v_{02}^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2$$

$$m_1 (v_{01}^2 - v_1'^2) = m_2 (v_2'^2 - v_{02}^2)$$

FOIL…

$$m_1 (v_{01} - v_1')(v_{01} + v_1') = m_2 (v_2' - v_{02})(v_2' + v_{02})$$

These cancel b/c they are equal…

So…

$$v_{01} + v_1' = v_{02} + v_2'$$

$$\underbrace{v_{01} - v_{02}}_{\text{approach}} = \underbrace{v_2' - v_1'}_{\text{departure}}$$

B. _______________________

1. In an _______________________, the __________________________________ an inelastic collision ___________________________ the kinetic energy ___________ the collision.
   a. What conditions can exist that would create an inelastic collision in an isolated system?

      1. There is still ____, such as an external friction force, because the system is isolated. There is also no sound wave transferring energy outside of the system.

      2. __________________________________, there ______________________________ between the objects causing them to _______________________.

      3. The ____, which involves ____ on the objects (deformation.) This may or may not result in the objects sticking together.

      - When the objects _______________________, the collision is said to be _______________________.

   b. What conditions can exist that would create ________________________________ _________________________?[4]

      1. The system may experience an ______________friction force.

      2. The collision may generate a ___________ wave carrying energy from the system to the environment.

2. Examples
   a. Example 11: (a) For example 10, what percentage of the original kinetic energy is conserved? (b) If the system were taken to be isolated (closed), what forms could the kinetic energy have taken? (c) If the system were taken to be open, what forms could the kinetic energy have taken?

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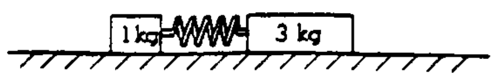
[4] Enduring Understanding 4.C: Interactions with other objects or systems can change the total energy of a system.

b. Example 12: Two symmetrically spherical objects move toward each other linearly. Object A has a mass of 1.0 kg and a velocity of 2.0 m/s. Object B has a mass of 2.0 kg and a velocity of 3.0 m/s. After the collision the 1.0 kg object has a velocity of -4.0 m/s. Consider the system to be isolated. Determine whether the collision is elastic, inelastic, or totally inelastic.

c. Example 13: A massless spring is between a 1 kilogram mass and a 3 kilogram mass as shown, but is not attached to either mass. Both masses are on a horizontal table. In an experiment, the spring is compressed. The 3 kilogram mass is then released and moves off with a speed of 10 meters per second. The 1kg mass also moves. There is friction with the table, but there is no friction between the spring and the blocks. (a) How much mechanical energy is released from the system in the form of thermal energy if the spring force constant is 2500N/m and the spring is initially compressed 0.09m? (b) Could you also then find the friction force? Provide a narrative answer.



Homework Riddle: Car A and Car B collide on a dynamics track in a physics lab, but not only is kinetic energy not conserved, momentum is not conserved, either. Describe both qualitatively what could account for this scenario and then generate a quantitative problem to describe the situation more fully. Be sure to solve for momentum loss and kinetic energy loss and account for where it went. *Hint: Revisit the definition of the law of conservation of momentum.* (*Learning Objective 5.D.1.2)*