

Unit 11: Electricity and Magnetism – Electric Circuits

OBJECTIVES

Big Idea 1: Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding 1.B: Electric charge is a property of an object or system that affects its interactions with other objects or systems containing charge.

Essential Knowledge 1.B.1: Electric charge is conserved. The net charge of a system is equal to the sum of the charges of all the objects in the system.

- a. An electrical current is a movement of charge through a conductor.
- b. A circuit is a closed loop of electrical current.

Learning Objective 1.B.1.1: The student is able to make claims about natural phenomena based on conservation of electric charge.

Learning Objective 1.B.1.2: The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits.

Big Idea 4: Interactions between systems can result in changes in those systems.

Enduring Understanding 4.E: The electric and magnetic properties of a system can change in response to the presence of, or changes in, other objects or systems.

Essential Knowledge 4.E.4: The resistance of a resistor and the capacitance of a capacitor can be understood from the basic properties of electric fields and forces as well as the properties of materials and their geometry.

- a. The resistance of a resistor is proportional to its length and inversely proportional to its cross-sectional area. The constant of proportionality is the resistivity of the material.
- b. The capacitance of a parallel plate capacitor is proportional to the area of one of its plates and inversely proportional to the separation between its plates. The constant of proportionality is the product of the dielectric constant, κ , of the material between the plates and the electric permittivity, ϵ_0 .
- c. The current through a resistor is equal to the potential difference across the resistor divided by its resistance.
- d. The magnitude of charge of one of the plates of a parallel plate capacitor is directly proportional to the product of the potential difference across the capacitor and the capacitance. The plates have equal amounts of charge of opposite sign.

Learning Objective 4.E.4.1: The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit based on the geometry of the circuit element and supported by scientific theories and mathematical relationships.

Learning Objective 4.E.4.2: The student is able to design a plan for the collection of data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors.

Learning Objective 4.E.4.3: The student is able to analyze data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors.

Essential Knowledge 4.E.5: The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

Learning Objective 4.E.5.1: The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

Learning Objective 4.E.5.2: The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

Learning Objective 4.E.5.3: The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors.

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.B: The energy of a system is conserved.

Essential Knowledge 5.B.3: A system with internal structure can have potential energy. Potential energy exists within a system if the objects within that system interact with conservative forces.

a. The work done by a conservative force is independent of the path taken. The work description is used for forces external to the system. Potential energy is used when the forces are internal interactions between parts of the system.

b. Changes in the internal structure can result in changes in potential energy. Examples should include mass-spring oscillators and objects falling in a gravitational field.

c. The change in electric potential in a circuit is the change in potential energy per unit charge.

Learning Objective 5.B.3.1: The student is able to describe and make qualitative and/or quantitative predictions about everyday examples of systems with internal potential energy.

Learning Objective 5.B.3.2: The student is able to make quantitative calculations of the internal potential energy of a system from a description or diagram of that system.

Learning Objective 5.B.3.3: The student is able to apply mathematical reasoning to create a description of the internal potential energy of a system from a description or diagram of the objects and interactions in that system.

Essential Knowledge 5.B.9: Kirchhoff 's loop rule describes conservation of energy in electrical circuits. [The application of Kirchhoff 's laws to circuits is introduced in Physics 1 and further developed in Physics 2 in the context of more complex circuits, including those with capacitors.]

- a. Energy changes in simple electrical circuits are conveniently represented in terms of energy change per charge moving through a battery and a resistor.
- b. Since electric potential difference times charge is energy, and energy is conserved, the sum of the potential differences about any closed loop must add to zero.
- c. The electric potential difference across a resistor is given by the product of the current and the resistance.
- d. The rate at which energy is transferred from a resistor is equal to the product of the electric potential difference across the resistor and the current through the resistor.
- e. Energy conservation can be applied to combinations of resistors and capacitors in series and parallel circuits.

Learning Objective 5.B.9.1: The student is able to construct or interpret a graph of the energy changes within an electrical circuit with only a single battery and resistors in series and/or in, at most, one parallel branch as an application of the conservation of energy (Kirchhoff 's loop rule). ¹

Learning Objective 5.B.9.2: The student is able to apply conservation of energy concepts to the design of an experiment that will demonstrate the validity of Kirchhoff 's loop rule ($\sum \Delta V = 0$) in a circuit with only a battery and resistors either in series or in, at most, one pair of parallel branches. ²

Learning Objective 5.B.9.3: The student is able to apply conservation of energy (Kirchhoff 's loop rule) in calculations involving the total electric potential difference for complete circuit loops with only a single battery and resistors in series and/or in, at most, one parallel branch. ³

Learning Objective 5.B.9.4: The student is able to analyze experimental data including an analysis of experimental uncertainty that will demonstrate the validity of Kirchhoff 's loop rule ($\sum \Delta V = 0$).

Learning Objective 5.B.9.5: The student is able to use conservation of energy principles (Kirchhoff 's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors.

Learning Objective 5.B.9.6: The student is able to mathematically express the changes in electric potential energy of a loop in a multi-loop electrical circuit and justify this expression using the principle of the conservation of energy

Learning Objective 5.B.9.7: The student is able to refine and analyze a scientific question for an experiment using Kirchhoff 's loop rule for circuits that includes determination of internal resistance of the battery and analysis of a non-ohmic resistor.

¹ This is the limit for AP Physics 1 but NOT for AP Physics 2.

² Same comment

³ Same comment

Learning Objective 5.B.9.8: The student is able to translate between graphical and symbolic representations of experimental data describing relationships among power, current, and potential difference across a resistor.

Essential Knowledge 5.C.3: Kirchhoff's junction rule describes the conservation of electric charge in electrical circuits. Since charge is conserved, current must be conserved at each junction in the circuit. Examples should include circuits that combine resistors in series and parallel. [Physics 1: covers circuits with resistors in series, with at most one parallel branch, one battery only. Physics 2: includes capacitors in steady-state situations. For circuits with capacitors, situations should be limited to open circuit, just after circuit is closed, and a long time after the circuit is closed.]

Learning Objective 5.C.3.1: The student is able to apply conservation of electric charge (Kirchhoff's junction rule) to the comparison of electric current in various segments of an electrical circuit with a single battery and resistors in series and in, at most, one parallel branch and predict how those values would change if configurations of the circuit are changed.⁴

Learning Objective 5.C.3.2: The student is able to design an investigation of an electrical circuit with one or more resistors in which evidence of conservation of electric charge can be collected and analyzed.⁵

Learning Objective 5.C.3.3: The student is able to use a description or schematic diagram of an electrical circuit to calculate unknown values of current in various segments or branches of the circuit.⁶

Learning Objective 5.C.3.4: The student is able to predict or explain current values in series and parallel arrangements of resistors and other branching circuits using Kirchhoff's junction rule and relate the rule to the law of charge conservation.

Learning Objective 5.C.3.5: The student is able to determine missing values and direction of electric current in branches of a circuit with resistors and NO capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule.

Learning Objective 5.C.3.6: The student is able to determine missing values and direction of electric current in branches of a circuit with both resistors and capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule.

Learning Objective 5.C.3.7: The student is able to determine missing values, direction of electric current, charge of capacitors at steady state, and potential differences within a circuit with resistors and capacitors from values and directions of current in other branches of the circuit.

⁴ This is the limit for AP Physics 1 but NOT for AP Physics 2.

⁵ Same comment

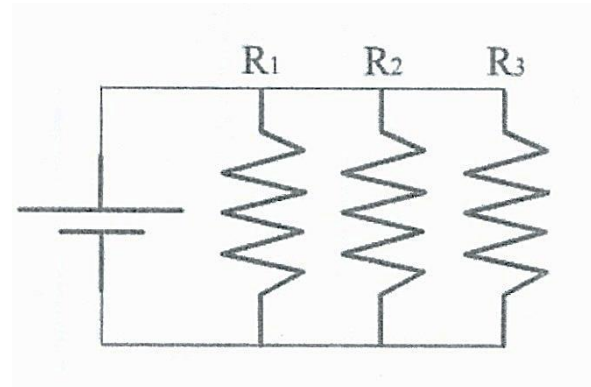
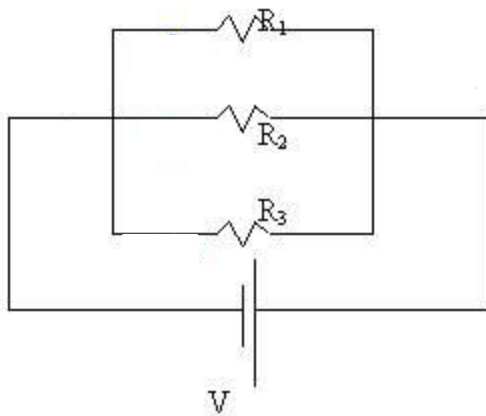
⁶ Same comment

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Schematic Diagrams

When drawing sketches of circuits, standard symbols are used so that people who are terrible artists can still draw a battery.

The order in which the circuit components are drawn is important. Drawing the circuit the way it is laid out is not important. The following diagrams represent the same circuit: Trace the paths to prove that they are the same.



Here are common symbols. We won't use them all, but we will use many of them.

Electric Circuit Symbols				
Conductor	Ground	Electric connection	No electric connection	Battery
Switch				
Fuse				
Capacitor				
Resistor (fixed)				
Potentiometer (variable resistor)				
Inductor				

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NOTES:

I. Electric Circuit Basics

A. _____:

1. _____: A circuit is a _____ of electrical current.

2. _____: An electric current is a _____ through a _____.

a. Electron flow vs. _____

1.) In solids, electrons carry charge in the opposite direction from the electric field.

2.) However, by convention, current is considered to be the _____ from _____ potential _____ potential along an electric field.

b. Details regarding current:

- Symbol:
- Formula:
- SI Unit:

3. Example 1: Current at a point in a circuit is measured to be 1.4 mA. How much charge passes through that point in the circuit every second? How many electrons is this?

B. _____

1. Conductors vary in their ability to permit current. Some oppose current more than others.

2. Details:

- Symbol:
- Formula:

where

- ρ is the _____ of the material measured in _____
- L is the length of the conductor.
- A is the cross-sectional area of the conductor.
- SI Unit:

3. Rate of heat production in a resistor

a. _____.

- 1.) Metals: Resistivity increases with temperature.
- 2.) Semiconductors: Resistivity decreases with temperature.
- 3.) Superconductors: Below a critical temperature, resistivity is zero.

b. As current flows, heat is produced, hence changing the resistivity.

- For many resistors, this is insignificant. We will deal mainly with these resistors.
- For some resistors, this effect is significant, and their resistance varies with voltage and current.

C. _____

1. For current to flow, a voltage source is needed.

2. The voltage source is sometimes called an _____, or _____, and is given the symbol _____.

D. _____

1. Relates current, resistance, and voltage

2. Formula:



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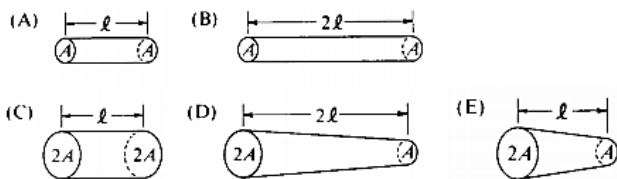
E. Electric power

- Formula:

(These second and third parts are not on the formula sheet but are easily derived by substitution using Ohm's law.)

F. Examples:

1. Example 2: The five resistors shown below have the lengths and cross-sectional areas indicated and are made of material with the same resistivity. Which has the greatest resistance?



2. Example 3: (a) What is the resistance of a particular length of standard 20-gauge copper wire that consumes 1000 W of power when connected to a 120 V source? (b) If 20-gauge wire has a cross-sectional area of $5.2 \times 10^{-7} \text{ m}^2$, and copper has a resistivity of $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$ at 20°C , how long is the wire? (c) How much current flows through it? (Solve for this two different ways.)

II. _____ and Components: The behavior of current in circuits depends on the arrangements of circuit components and the properties of the components themselves.

A. There are three types of circuit geometry.

1. _____

a. Defining characteristic: _____

b. Properties:

1.) Current:

a) _____ is the _____.

b) The current through a resistor is equal to the potential difference across the resistor divided by its resistance. (In other words, _____.)

2.) Resistance:

a) When multiple resistors are combined in series, the _____ simply _____ to what is called an _____.⁷

b) Formula:⁸

3.) Voltage:

a) The _____ across each resistor is _____ portion of the total resistance. (i.e., A resistor with more resistance requires more voltage to push current through it.)

- Another way to think of this: The voltage used to push a certain amount of current through a resistor can be found by Ohm's law where $\Delta V = IR$.
 - For example: To push a lot of current takes a lot of voltage.
To push current through a lot of resistance takes a lot of voltage.

⁷ In other words, if there were only one resistor in the circuit instead of multiple resistors, what would the resistance of that one resistor have to be to equal the others?

⁸ Please look at your formula sheet for the correct notation. The equation function wasn't allowing the correct input of the letter *i* under the sigma. That will show up in later formulas, as well.



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b) _____ describes _____
 _____ in electrical circuits. _____
 _____ (i.e., voltage
 added to the circuit combined with voltage used by components combines
 to zero.)

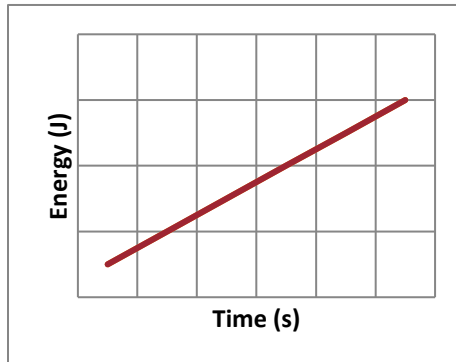
- Energy changes in simple electrical circuits are conveniently represented in terms of energy change per charge moving through a battery and a resistor. Since electric potential difference times charge is energy, and energy is conserved, the sum of the potential differences about any closed loop must add to zero.
 - Remember that Voltage is Joules/Coulomb.
 - Remember that current is the rate of charge flow.
- Note that this runs _____.
 - Therefore when a battery's polarity matches the overall polarity of the circuit, that battery adds emf. However, if a battery's polarity is reversed, compared to the overall polarity, that reversed battery uses voltage.
- This _____, not just series circuits.

4.) Power: Power dissipation can be found from I , R , and ΔV . Think conceptually:

- For example: A resistor with a lot of current and a lot of potential difference across it would have (a lot/a little) power dissipation.

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- c. Example 4: A series circuit has a 12V battery and three resistors as follows: 320Ω , 450Ω , and 115Ω . (a) Sketch the circuit using standard schematic symbols. (b) Calculate the equivalent resistance of the circuit and the voltage, current, and power dissipation of each resistor. (c) If the following shows a graph of the energy used by the 115Ω resistor as a function of time, sketch what the curves would look like for the other two resistors in the same time period. (You can do this on the same graph.)



2. _____

a. Defining characteristic: _____;
current _____ at _____

b. Properties:

1.) Current:

a) Current depends on the total resistance in each branch and the potential difference across each branch, according to Ohm's Law.

b) Therefore, _____.

c) _____ describes the _____
_____ in electrical circuits. Since charge is conserved, current must be conserved at each junction in the circuit.⁹

- In other words, _____
_____ because the same amount of charge per second that enters must come out. This relates to the law of conservation of charge.

2.) Resistance:

a) _____ for the entire circuit actually _____
when resistors are added in parallel.

b) Formula:

3.) Voltage: _____ across all resistors branching
_____.

4.) Power dissipation can be found from I , R , and ΔV . Think conceptually:

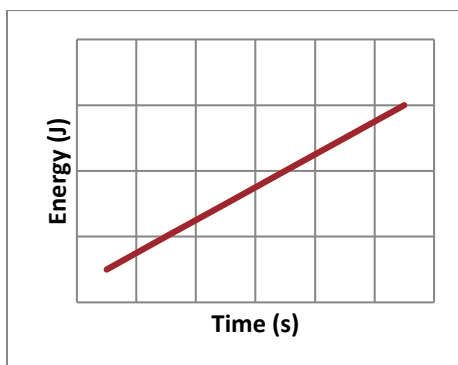
- For example: A branch with a lot of current and the same potential difference across it as the other branches would have (more/less) power dissipation than the other branches.

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⁹ Big idea 5, anyone? ☺

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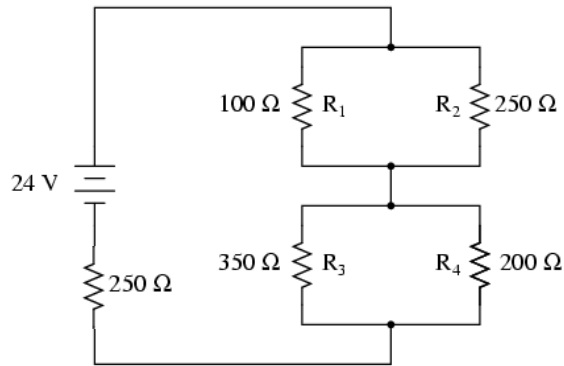
- c. Example 5: Repeat example 4 for a parallel circuit. Compare the values to those in a series circuit. Also repeat exercise (c) by sketching the energy v. time curves for the larger two resistors.



3. _____ contain series and parallel components. Each component must be handled before addressing the circuit as a whole. Kirchoff's rules guide us.

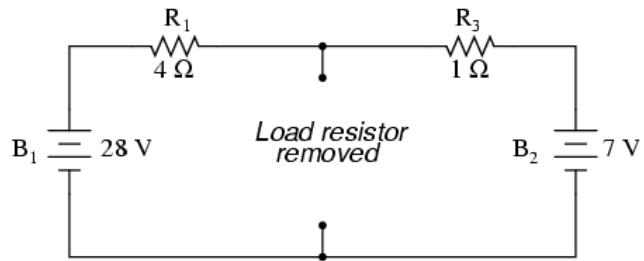
- a. Example 6: For the following circuit, (a) calculate the equivalent resistance of the circuit, the total current and power drawn by the circuit, and the voltage, current, and power dissipation of each resistor.

A series-parallel combination circuit



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- b. Example 7: For the following circuit, calculate the equivalent resistance of the circuit, the total current and power drawn by the circuit, and the voltage, current, and power dissipation of each resistor.



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B. Ideal and real batteries

1. _____ are fictitious batteries that have _____ themselves and therefore put out 100% of their voltage into the circuit.
2. _____ do have _____¹⁰; therefore they _____ to push current into the circuit.
 - a. The voltage emerging from such a voltage source is called _____.
 - b. Internal resistance can vary, decreasing with greater current.
 - c. Helpful formula not on the formula sheet:

where _____ is the internal resistance of the voltage source.

- d. Example 8: Calculate the terminal voltage of a battery of specified emf and internal resistance from which a known current is flowing: An old 1.5V battery has an internal resistance of $2.0\ \Omega$. What is the terminal voltage when 6mA flow out of it? What's the resistance in the main part of the circuit?

¹⁰ Batteries' internal resistance comes from energy required for the chemical reactions; generators' internal resistance comes from the fact that there are wires and other components within a generator.

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C. Voltmeters and ammeters

1. _____ measure voltage.

a. Symbol:

b. Voltmeters must be _____ but _____
_____ in order to measure the potential
difference between two points in a circuit without altering the circuit. Therefore they
must have _____.

c. Sketch:

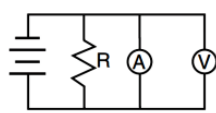
2. _____ measure current.

a. Symbol:

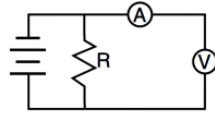
b. Ammeters must be _____ but must _____
_____ to the circuit so as not to alter the
circuit.

c. Sketch:

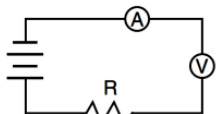
3. Example 9: Which of these is correct?



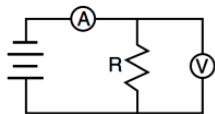
(1)



(3)



(2)

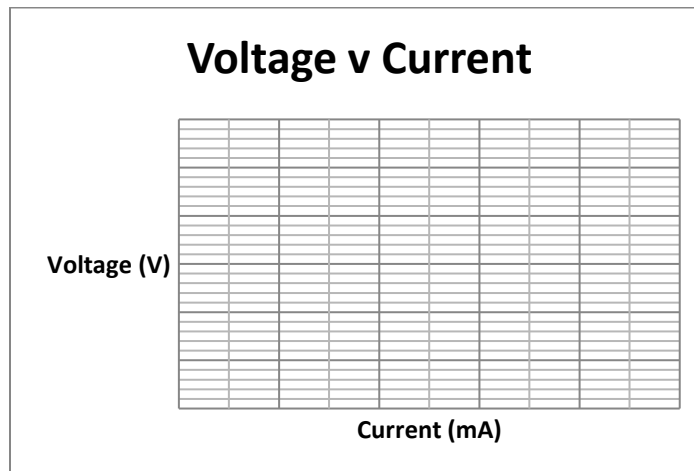


(4)

D. Ohmic and Non-Ohmic Resistors

1. _____ follow Ohm's law.
2. _____ vary their resistance as voltage and current change. They _____. They belong to the _____ of resistors.
3. Example 10: Resistor A and resistor B are tested individually in a circuit with a power supply that can vary its voltage. The following readings were taken. (a) Determine graphically whether each resistor is ohmic or non-ohmic. (b) You later find out that there was enough uncertainty in the ammeter to put your conclusion in doubt. What's the minimum uncertainty that would create this doubt in your mind?

Resistor A		Resistor B	
Voltage (V)	Current (mA)	Voltage (V)	Current (mA)
1	0.010	1	0.010
2	0.020	2	0.013
3	0.030	3	0.015
4	0.040	4	0.016
5	0.050	5	0.017



III. Capacitors in circuits

- A. _____ when connected to a voltage source and can be combined in the same combinations as resistors.
- B. Purpose: _____ (Resistor-Capacitor) go through a capacitor charging phase and a capacitor discharging phase when the capacitor reaches its threshold. Usually while charging there is not enough current to run the appliance, but during discharge, the appliance works.
Examples: pacemakers, intermittent windshield wiper blades

C. _____

1. Formula:

- Derivation: $V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = q\left(\frac{1}{C_s}\right)$ so multiple capacitors in series can be replaced by $\frac{1}{C_s}$

2. _____ (_____) is the _____ in all capacitors in series¹¹;
_____ is _____.

- $V \propto \frac{Q}{C}$ so as _____, _____
in _____ proportionally.

○ *Steps for solving:*

- Find C_s
- Use $Q = C_s V$ to find Q , which is the same in each capacitor.
- Use $V = \frac{Q}{C}$ with Q and each capacitor's capacitance to find V in each.

3. Example 11: Three capacitors (2.0, 4.0, and 6.0 μF) are wired in series with a 12V battery.
Find the equivalent capacitance, the charge stored in each, and the voltage used by each.

¹¹ Capacitors in series charge each other by induction! Think about it! ☺

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D. _____

1. Formula:

2. _____ is the _____ in all capacitors in parallel; _____
(_____) is divided.

- $Q \propto CV$ so as capacitance goes up, current goes up proportionally.
- *Steps for solving:*
 - Find C_p
 - Use $Q = CV$ with V (equaling the emf of the voltage source) and each capacitor's capacitance to find Q in each.

3. Example 12: Repeat example 11 with the capacitors in parallel.

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(A long overdue) Intermission: Essential Knowledge 5.C.3 indicates that “Physics 2: includes capacitors in steady-state situations. For circuits with capacitors, situations should be limited to open circuit, just after circuit is closed, and a long time after the circuit is closed.” This means that we can ignore the period during which the capacitors are in the process of being charged, which can take some time. For us, capacitors will either be just starting to charge or fully charged.

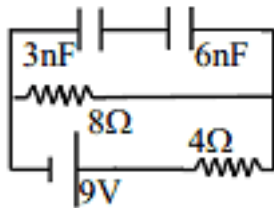
E. Calculating R-C Circuits

1. The trick:

a. When _____ capacitors act as a short circuit. All current runs through them, and the _____ across them is _____. Why does this make sense?

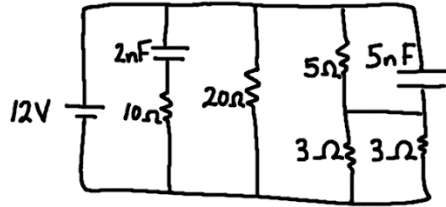
b. Once _____, charge (i.e., _____) _____ to that capacitor, essentially breaking that branch of the circuit. Now _____ exists across the capacitor.

2. Example 13: The circuit shown below has been connected for a long time. Calculate (a) the current in the 4Ω resistor, and (b) the potential difference across the 8Ω resistor, and (c) the charge stored on one plate of each capacitor, and (d) the voltage in each capacitor.



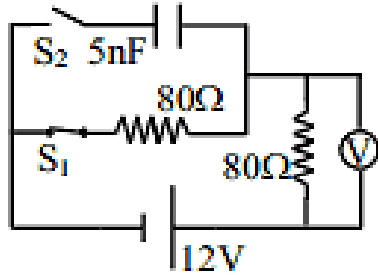
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3. Example 14: This circuit has been connected for a long time. Solve for the current and voltage in all the resistors below and the voltage and charge in each capacitor...and don't be intimidated. 😊



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4. Example 15: Consider the circuit shown below. Initially switch S_1 is closed, and switch S_2 is open. (a) Find the reading on the voltmeter. (b) Switch 1 is then opened, and S_2 is closed. A long time later find the reading of the voltmeter and (c) the charge on the capacitor.

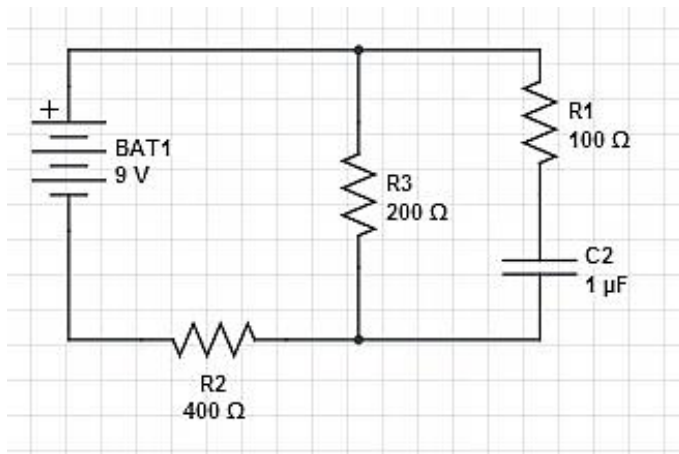
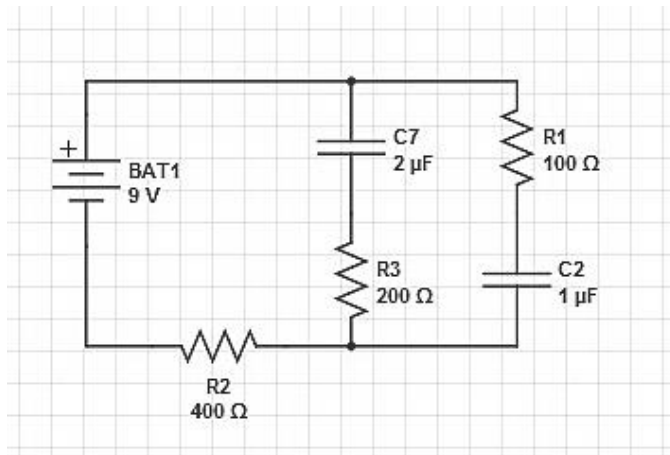


Answer to the uncertainty question from example 10:

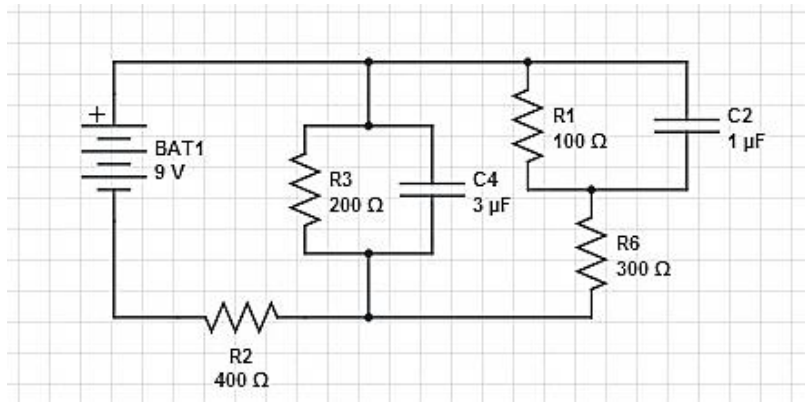
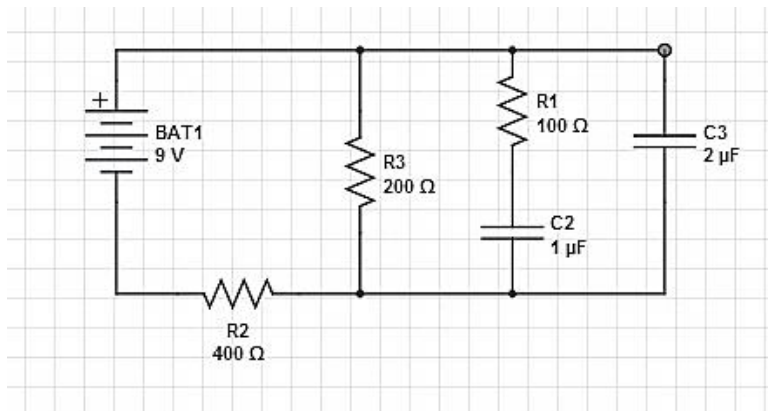
The slope of Voltage v Current gives us resistance via Ohm's law. The slope for resistor A is constant and equals 100, which matches the individual calculations for each measurement for resistor A (i.e., if you calculate the resistance for all five trials using Ohm's law, the resistance equals 100 Ω .) As the slope varies for resistor B, it is non-ohmic. The slope of the best fit line is approximately 550, but by individual calculations, the resistances for the five trials are 100 Ω , 154 Ω , 200 Ω , 250 Ω , and 294 Ω respectively. The largest discrepancy from the slope of the best fit line is the first trial. If the resistance is truly 550 Ω , it would have drawn 0.002 mA of current. Instead it drew 0.010 mA of current. Therefore if the uncertainty of the ammeter were at least ± 0.008 mA, the result could stem from systematic error, and resistor B may be ohmic after all.

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Extra circuits to practice on...Answers are on the last page.



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Problem 1

	R (Ohms)	I (A)	V (V)
	400	0	0
	200	0	0
	100	0	0
Total	infinite	0	9

	C (μ F)	Q (μ C)	V
	2	18	9
	1	9	9
Total	irrelevant	irrelevant	9

Problem 2

	R (Ohms)	I (A)	V (V)
	400	0.015	6
	200	0.015	3
	100	0	0
Total	600	0.015	9

	C (μ F)	Q (μ C)	V
Total	1	3	3

Problem 3

	R (Ohms)	I (A)	V (V)
	400	0.015	6
	200	0.015	3
	100	0.015	0
Total	600	0.015	9

	C (μ F)	Q (μ C)	V
	1	3	3
	2	6	3
Total	irrelevant	irrelevant	3

Problem 4

	R (Ohms)	I (A)	V (V)
	100	0.006	0.60
	400	0.017	6.75
	200	0.011	2.25
	300	0.006	1.80
Total	533	0.017	9.00

	C (μ F)	Q (μ C)	V
	3	6.75	2.25
	1	0.60	0.60
Total	irrelevant	irrelevant	N/A