

GUIDED NOTES

Unit 6: Newtonian Mechanics – Circular and Rotational Motion

OBJECTIVES

We will continue to reinforce the concepts explored previously.

NOTE: There are no AP Physics 1 or AP Physics 2 learning objectives that focus *solely* on some of the topics (namely circular motion) covered in this unit. However, the topics we will cover appear on the formula sheet and are embedded within the following learning objectives.

Big Idea 3: The interactions of an object with other objects can be described by forces.

Enduring Understanding 3.F: A force exerted on an object can cause a torque on that object.

Essential Knowledge 3.F.1: Only the force component perpendicular to the line connecting the axis of rotation and the point of application of the force results in a torque about that axis.

a. The lever arm is the perpendicular distance from the axis of rotation or revolution to the line of application of the force.

b. The magnitude of the torque is the product of the magnitude of the lever arm and the magnitude of the force.

c. The net torque on a balanced system is zero.

Learning Objective 3.F.1.1: The student is able to use representations of the relationship between force and torque.

Learning Objective 3.F.1.2: The student is able to compare the torques on an object caused by various forces.

Learning Objective 3.F.1.3: The student is able to estimate the torque on an object caused by various forces in comparison to other situations.

Learning Objective 3.F.1.4: The student is able to design an experiment and analyze data testing a question about torques in a balanced rigid system.

Learning Objective 3.F.1.5: The student is able to calculate torques on a two-dimensional system in static equilibrium by examining a representation or model (such as a diagram or physical construction.)

Essential Knowledge 3.F.2: The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis.

a. Rotational motion can be described in terms of angular displacement, angular velocity, and angular acceleration about a fixed axis.

b. Rotational motion of a point can be related to linear motion of the point using the distance of the point from the axis of rotation.

c. The angular acceleration of an object or rigid system can be calculated from the net torque and the rotational inertia of the object or rigid system.

Learning Objective 3.F.2.1: The student is able to make predictions about the change in the angular velocity about an axis for an object when forces exerted on the object cause a torque about that axis.

Learning Objective 3.F.2.2.: The student is able to plan data collection and analysis strategies designed to test the relationship between a torque exerted on an object and the change in angular velocity of that object about an axis.

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Essential Knowledge 3.F.3: A torque exerted on an object can change the angular momentum of an object.

- a. Angular momentum is a vector quantity, with its direction determined by a right-hand rule.
- b. The magnitude of angular momentum of a point object about an axis can be calculated by multiplying the perpendicular distance from the axis of rotation to the line of motion by the magnitude of linear momentum.
- c. The magnitude of angular momentum of an extended object can also be found by multiplying the rotational inertia by the angular velocity.
- d. The change in angular momentum of an object is given by the product of the average torque and the time the torque is exerted.

Learning Objective 3.F.3.1: The student is able to predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum.

Linear Objective 3.F.3.2: In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object.

Learning Objective 3.F.3.3: The student is able to plan data collection and analysis strategies designed to test the relationship between torques exerted on an object and the change in angular momentum of that object.

Big Idea 4: Interactions between systems can result in changes in those systems.

Enduring Understanding 4.D: A net torque exerted on a system by other objects or systems will change the angular momentum of the system.

Essential Knowledge 4.D.1: Torque, angular velocity, angular acceleration, and angular momentum are vectors and can be characterized as positive or negative depending upon whether they give rise to or correspond to counterclockwise or clockwise rotation with respect to an axis.

Learning Objective 4.D.1.1: The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system.

Learning Objective 4.D.1.2: The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data.

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Essential Knowledge 4.D.2: The angular momentum of a system may change due to interactions with other objects or systems.

- The angular momentum of a system with respect to an axis of rotation is the sum of the angular momenta, with respect to that axis, of the objects that make up the system.
- The angular momentum of an object about a fixed axis can be found by multiplying the momentum of the particle by the perpendicular distance from the axis to the line of motion of the object.
- Alternatively, the angular momentum of a system can be found from the product of the system's rotational inertia and its angular velocity.

Learning Objective 4.D.2.1: The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems.

Learning Objective 4.D.2.2: The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems.

Essential Knowledge 4.D.3: The change in angular momentum is given by the product of the average torque and the time interval during which the torque is exerted.

Learning Objective 4.D.3.1: The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum.

Learning Objective 4.D.3.2: The student is able to plan a data collection strategy designed to test the relationship between the change in angular momentum of a system and the product of the average torque applied to the system and the time interval during which the torque is exerted.

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A: Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2: For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

Enduring Understanding 5.E: The angular momentum of a system is conserved.

Essential Knowledge 5.E.1: If the net external torque exerted on the system is zero, the angular momentum of the system does not change.

Learning Objective 5.E.1.1: The student is able to make qualitative predictions about the angular momentum of a system for a situation in which there is no net external torque.

Learning Objective 5.E.1.2: The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero. (Do rigid compound pulley problems.)

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Essential Knowledge 5.E.2: The angular momentum of a system is determined by the locations and velocities of the objects that make up the system. The rotational inertia of an object or system depends upon the distribution of mass within the object or system. Changes in the radius of a system or in the distribution of mass within the system result in changes in the system's rotational inertia, and hence in its angular velocity and linear speed for a given angular momentum. Examples should include elliptical orbits in an Earth-satellite system. Mathematical expressions for the moments of inertia will be provided where needed. Students will not be expected to know the parallel axis theorem.

Learning Objective 5.E.2.1: The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses.

NOTES:

I. Uniform Circular Motion: Objects traveling in _____ at a _____

A. Introduction:

1. _____ of an object traveling in uniform circular motion...

a. _____.

b. _____ is _____ to the circle. Therefore the velocity is often called _____.

1.) Therefore _____ (and, _____, _____) is _____.

2.) Therefore _____ the object is _____...

2. _____

a. The _____ at which an object traveling in uniform circular motion _____

- If it's changing direction more quickly, it is traveling faster around the circle with a larger acceleration.

b. Centripetal acceleration is _____, _____ the _____.

c. Because the object is accelerating, by Newton's second law there _____ that is being applied to the object...

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d. Examples 1 & 2:

1.) A gear turns 35 times a second. What are the frequency and period? Write these in sentence form: *The frequency is _____, which means the gear turns through _____ turns in one second. The period is _____, which means it takes _____ for the gear to turn once.*

2.) A carousel takes 20 seconds to rotate. What are the frequency and period? *The frequency is _____, which means the carousel turns through _____ turns in one second. The period is _____, which means it takes _____ for the carousel to turn once.*

7. Formulas:

Value	Formula	Units	Direction
Tangential Velocity			
Centripetal Acceleration			
Centripetal Force			

8. Determine the components of the velocity and acceleration vectors at any instant and sketch or identify graphs of these quantities²: <http://www.walter-fendt.de/ph14e/circmotion.htm>

² A quality objective from the AP Physics B curriculum

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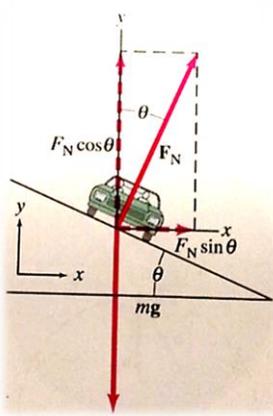
B. The Horizontal Circle

1. Example 3 – Horizontal circle with no incline: A 35 kg child sits on a carousel horse so that her center of mass is 7.5 m from the center of the carousel. The carousel rotates once in 20 seconds. (a) What is the child's tangential velocity and centripetal acceleration? (b) Do you consider this to be a fast velocity? Why or why not? (c) What is the force that causes this acceleration? (d) What is the source of this force? (e) Identify the system by describing the force as internal or external to explain the motion. (f) Varying only one variable at a time, come up with two ways to increase the little rider's tangential velocity so that the ride is less not awesome.

2. Example 4 – Horizontal circle with no incline: A car rounds a curve on a flat road of radius 50m at a speed of 14m/s. Will the car make the turn or skid out if (a) the pavement is dry and the coefficient of static friction is 0.60 (b) the pavement is icy and the coefficient of static friction is 0.25? (c) Why does this question involve static friction instead of kinetic friction?

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3. Example 5 – Horizontal circle, banked curve (inclined): The normal force will now have a horizontal component that is in the same direction as F_c , thus giving more ability to apply centripetal force than with friction alone. The angle at which no friction is required is found by...³



$$\vec{F}_c = \vec{F}_N \sin\theta = m\vec{a}_c = m\frac{\vec{v}^2}{r}$$

$$\left(\frac{m\vec{g}}{\cos\theta}\right) \sin\theta = m\frac{\vec{v}^2}{r}$$

$$\vec{g}\tan\theta = \frac{\vec{v}^2}{r}$$

$$\tan\theta = \frac{\vec{v}^2}{\vec{g}r}$$

(This formula is not on the formula sheet.)

- Question: In example 4 what angle would prevent the car from skidding on the wet pavement?

³ Note: This is different from an inclined plane where an object is sliding downward. In that case, $\vec{F}_N = mg\cos\theta$. However, in this case, $\vec{F}_N \geq mg\cos\theta$ because the normal force is both preventing the car from falling through the ground (i.e., counteracting gravity) AND it's overcoming inertia to provide some of the centripetal force. If this confuses you and you care that it confuses you, I can point you to a book (Giancoli) that gives a more thorough explanation of the geometry. However, it is beyond the scope of this course.

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C. The Vertical Circle

1. Example 6 – Vertical circle in which tension creates \vec{F}_c : A mass swings vertically on a string so that _____...

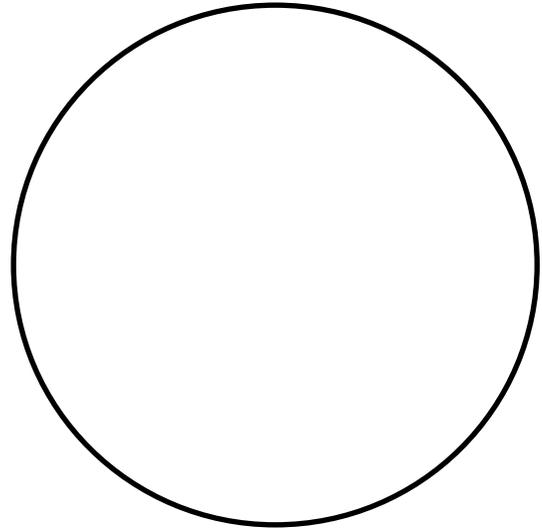
- Top:
- Sides:
- Bottom:

Thought process...

At the top, gravity helps provide centripetal force, so the tension can be _____.

In the middle, the tension is perpendicular to gravity. Gravity doesn't work with or against the centripetal force, so...

At the bottom, the tension has to do two things: Overcome gravity and provide centripetal force. Tension must be _____.



- A 0.300kg ball on the end of a string is revolved at a uniform rate in a vertical circle of radius 85.0cm with a speed of 4.15m/s. What is the tension in the string at all four points?

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2. Example 7 – Vertical circle in which in which an object moves *within* a rigid vertical loop:

When mass moves within a rigid vertical loop ...

_____ force...

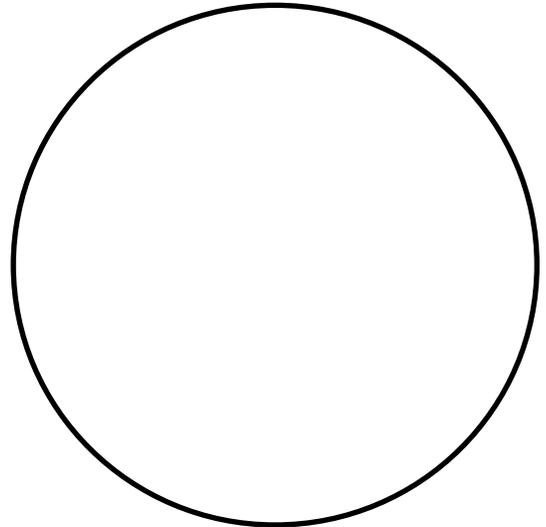
- Top:
- Sides:
- Bottom:
- This follows the same pattern as tension.

Thought process...

At the top, gravity helps provide centripetal force, so the normal force can be _____.

In the middle, the normal force is perpendicular to gravity. Gravity doesn't work with or against the centripetal force, so...

At the bottom, the normal force has to do two things: Overcome gravity and provide centripetal force. Normal force must be _____.



3. Example 8 – Vertical circle in which in which an object moves *on top of* a rigid vertical

loop: When mass moves over the top of a rigid vertical loop, _____

centripetal force. If the required centripetal force is larger than the object's weight, the object lifts.

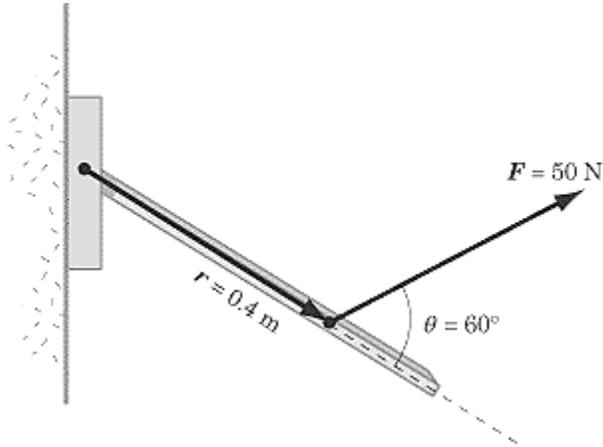
- Question: What is the maximum velocity a vehicle can have as it travels over the circular crest (radius 15.0 m) of a hill without the vehicle leaving the road? How would you use this to set a speed limit?

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II. Rotational Motion

A. _____: The _____ of a _____ to _____

1. The line connecting the axis of rotation and the point of application of the force is called the _____.
2. Only the _____ to lever arm results in a torque about that axis.



3. Symbol:

4. Formula:

where...

- r is the length of the lever arm
- \vec{F}_\perp is the force component perpendicular to the lever arm.⁴

5. SI Unit: _____

6. A vector...

- a. _____ torque is _____.
- b. _____ torque is _____.

⁴ Note that on the formula sheet $\sin\theta$ is included in the formula. You'll do better under all circumstances if you're able just to figure it out using vectors.

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7. Example 9: A biceps muscle exerts a vertical force of 700N on a lower arm. (a) Assuming the bicep is attached 5cm from the elbow joint and that it makes an angle of 30° to the arm, what is the torque the bicep applies? (b) What happens to the torque as the arm moves upward? (c) Chimpanzees have about $1/3$ the muscle mass of a human, but they are much stronger in some movements. Account for this.⁵

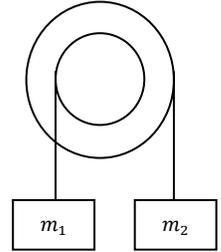
B. Rotational and _____ considerations

1. Rotational statics: The net torque on a balanced system is zero. No change in rotational state occurs.
2. The presence of a net torque along any axis will cause a rigid system to change its rotational motion or an object to change its rotational motion about that axis.
3. _____ in the case of a rotational system _____.
the _____ of the _____. This will be straight-line motion, projectile motion, or otherwise depending on external forces.
Generate an example: (For example, a car tire rotating while the car moves forward)

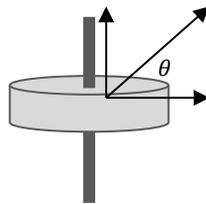
⁵ This terrific problem flagrantly stolen from a textbook.

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4. Example 10 – Torque created by gravity/torque on a compound wheel: Two thin, cylindrical, rigid wheels of radii r_1 (inner) and r_2 (outer) are firmly attached to each other on an axle that passes through the center of each. (a) Calculate the net torque on this compound wheel due to the two masses shown, m_1 and m_2 . Put the answer in terms of m , g , and r . (b) If the system is held in translational equilibrium by a cord from above, what is the tension in the cord?



5. Example 11 – Torque on a two-dimensional system: A wire pulls upward on a disc of radius 0.50m with a force of 25 N at an angle of 60° above the horizontal, as shown. (a) What's the torque on the disc? (b) What could the component of the force that does not contribute to the torque do to the disc-axle system? (c) Describe quantitatively an additional upward or downward force that could prevent the disc from turning.



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C. Rotational Motion: Rotational _____ can be described in terms of angular displacement, angular velocity, and angular acceleration about a fixed axis.

1. The _____ for angles when considering rotational (angular) motion is the _____ where 1 radian equals the angle at which the arc length equals the radius. The conversion from degrees to radians can be derived from the circles below.⁶

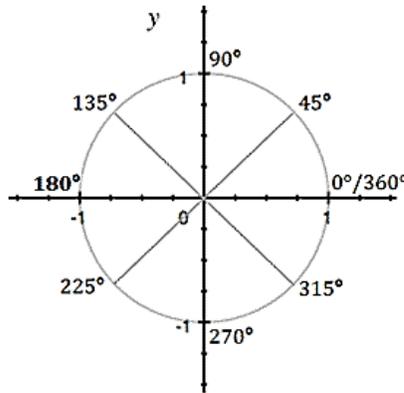


Figure 1: Unit circle measured in degrees.

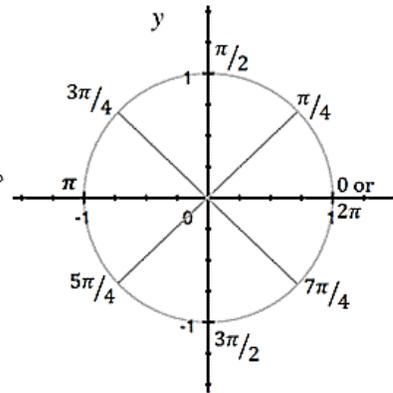


Figure 2: Unit circle measured in radians.

2. _____:
 - a. The angle through which a disc turns
 - b. Symbol:

3. _____:
 - a. The rate at which an disc undergoes angular displacement
 - b. Formula:

 - c. SI units:

4. _____:
 - a. The rate at which an disc undergoes a change in angular velocity
 - b. Formula:

 - c. SI units:

⁶ 360° equals 2π rad, so 1 rad equals 57.3°.

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5. Kinematic equations for rotational motion mirror those for linear motion. These are the only ones shown on your formula sheet, but all four kinematic equations have angular parallel equations. Sweet!

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

6. Example 12: An amusement park ride consists of a rotating disk of radius 15 m. From rest the ride gets up to a maximum clockwise angular velocity that allows it to complete one full turn in 12 seconds. (a) What is its maximum angular velocity in SI units? (b) What is this in degrees per second? (c) If it took 68 seconds to reach this maximum angular velocity, what is its angular acceleration in SI units while it is speeding up? (d) What was its angular displacement during the acceleration period? (e) How many turns is this? (f) Is this a thrilling ride? (Honest question...Look for meaning in the numbers.)

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7. _____ (meaning the motion of a point mass as part of a rotating system) can be _____ of the point using the distance of the point from the axis of rotation. (This relates back to the tangential velocity circular motion, of course. ☺)

a. The linear velocity of the point is⁷

Converted to degrees/second, we get tangential velocity.

b. This gets trickier...When a rotating disc changes its angular speed, the points on the disc undergo _____ that are _____ to each other.

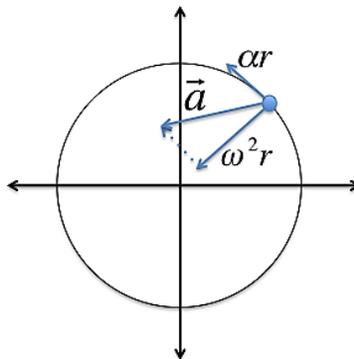
1.) One acceleration is good old fashioned _____ acceleration to keep the object traveling in a circle. This is due to changing direction.

2.) The other is _____ when the point speeds up or slows down _____.

This can be found by

3.) Both of these often happen at the same time. Generate some examples of this happening in real life:

4.) Since these are at right angles to each other, the total acceleration of the point can be found via the Pythagorean theorem.⁸



⁷ This can be found using the conversion between radians and degrees in the unit circle. This isn't on the formula sheet, but you should be able to figure it out conceptually.

⁸ It is a messy mess. Don't worry about it quantitatively.

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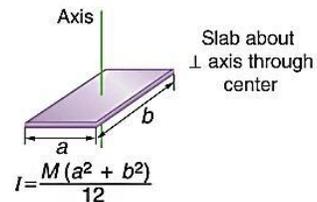
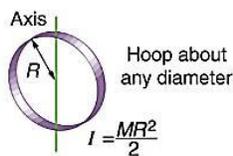
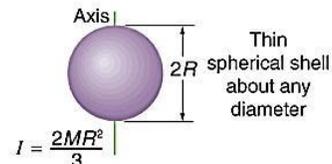
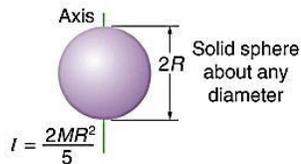
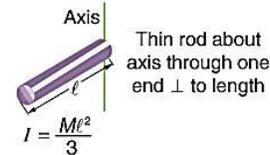
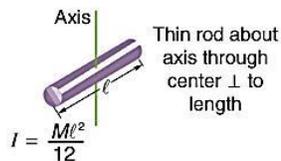
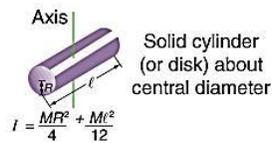
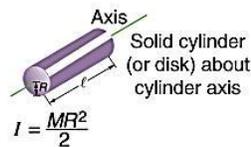
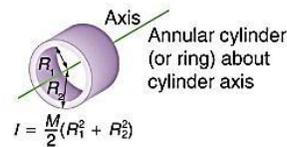
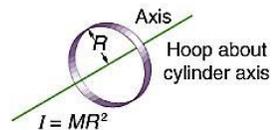
- 5.) Example 13: A 0.5kg bucket hanging above a well from a frictionless pulley descends due to gravity. The pulley is rigid, massless, and has a radius of 0.25 m.
- (a) What is the tangential acceleration of a point on the outside of the pulley?
- (b) If the bucket falls for 3.5 seconds, what's the increase in angular velocity of the disc? (c) Is this fast or slow? (Justify your answer.) (d) What is the tangential velocity of the point?

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D. _____

1. In plain language, it's the tendency of an object to _____. In physics lingo, however it is also called...
2. _____⁹

- a. This determines how much torque is required to generate an angular acceleration, just like mass determines how much force is required to accelerate an object translationally.
- b. Symbol:
- c. SI Unit:
- d. The _____ and the _____ of the object determines the moment of inertia. Each shape and mass distribution has a unique formula based on the mass, M , and the radius, R . You will be provided with the necessary formula if given a problem involving moment of inertia. Here are some common examples.



⁹ A very poetic name. Feel free to use it for your next rock band.

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3. A rotating object's _____ can be found by taking angular velocity and moment of inertia into account.¹⁰ This rotational kinetic energy¹¹ is found by

4. _____ causes _____ in _____ to the object's _____ according to the relationship

5. Example 14: Return to example 13, but assume instead that the pulley exists in the real world. Therefore it has mass (2.5 kg, in fact) and experiences kinetic friction as it falls due to the rope rubbing on the pulley. How does the tangential acceleration change? Finally, why didn't your teacher ask you to find the new change in angular velocity?

¹⁰ Remember that mass is a measure of inertia...inertial mass? Remember the good old days?

¹¹ This is nowhere in the curriculum BUT it *is* in a sample problem given with the curriculum, and it *is* on the formula sheet. We all appreciate the clarity of the document. ☺

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E. _____

1. This is a measure of the _____ of a rotating object _____.

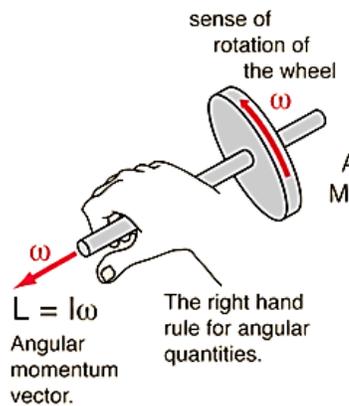
2. Details:

a. Symbol:

b. Formula:

c. SI Unit: _____ (Note that it is not $\text{kg}\cdot\text{m}/\text{s}^2$.)

d. Angular momentum is a _____ quantity whose direction follows a really weird _____ as follows:



OK, so your fingers point in the direction the wheel is *actually* spinning, which suggests that this is the direction of the angular velocity vector and the angular momentum vector. If you thought this very logical thought, you would be wrong. (This is when physics laughs maniacally because it is blatantly just messing with us.) The angular velocity and the angular momentum (and, in fact, the torque, too) aren't considered to be in the actual direction of the rotation. They are considered to point parallel to the rotational axis. Keep this in mind. It will mess with us very soon. ☺

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3. Analog to the impulse-momentum theorem: A _____ exerted _____ by other objects or systems will _____ of the system according to the relationship
- a. The quantity $\tau\Delta t$ is called _____.
- b. Example 15: A 6.2 kg, 0.35m radius potter's wheel spins at 6.28 rad/s. The potter slaps the edge of the wheel perpendicular to the radius with a force of 45 N in order to increase the wheel's angular velocity. The collision between the hand and the wheel takes 0.091 seconds. The moment of inertia of a uniform solid disk that turns around a central axis is $I = \frac{1}{2}MR^2$. (a) Describe the change in angular momentum. (b) What is the wheel's new angular velocity? (c) What is the angular acceleration? (d) What is she making?
- c. Example 16: In an old mill, a rigid paddlewheel of mass 350 kg and radius 4.5 m has slats onto which water can fall perpendicularly from a chute. When the chute is opened, water exerts a force of 250 N during a collision that lasts 0.55 seconds when the water hits the first slat. Simultaneous to the first hit of water from the chute, the slat at the bottom receives 190 N of resistive force from the river below. (a) What is the net angular impulse? (b) What is the change in angular momentum?

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4. Angular momentum of a _____:
- a. _____ object (or “_____”) can be considered to have _____ that _____ the overall _____ of the system.
- 1.) If we _____ the _____ of the object (kg·m/s) _____ to the line of motion of the object, we arrive at the amount of angular momentum that particle contributes to the whole system.

This is not on the formula sheet.

(Follow the units: pR gives _____.)

- 2.) For example, each person on Earth contributes to the mass of Earth at its surface. Each buried rock contributes to the mass of Earth at its radial position. All together, the linear momenta of these objects as they cruise in a circular motion around Earth’s axis contributes to the angular momentum of Earth. Let’s look at how much *you* contribute!

Example 17: Let’s look at how much YOU contribute to Earth’s angular momentum. Find (a) your tangential velocity as you cruise on Earth’s surface; (b) your linear momentum (c) the amount of momentum you contribute to angular momentum of the Earth-you system; and (c) the total angular momentum of the Earth-you system.

- Earth’s angular velocity: 7.27 E-5 rad/s
- Earth’s mass: 5.97 E24 kg
- Earth’s radius: 6.37 E6 m
- Your mass (pounds/2.2): _____ kg

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- b. Each object (or “particle”) also has its own angular momentum that contributes to the angular momentum of the whole. We’ll examine this in example 20.

Big Idea 5: Changes that occur as a result of interactions are constrained by conservation laws.

Enduring Understanding 5.A: Certain quantities are conserved, in the sense that the changes of those quantities in a given system are always equal to the transfer of that quantity to or from the system by all possible interactions with other systems.

Essential Knowledge 5.A.2: For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

Enduring Understanding 5.E: The angular momentum of a system is conserved.

5. _____.

- a. What constitutes an isolated system here?

- b. Example 18: Why does an ice skater speed up when he brings his arms and legs in? What does he do to slow down at the end of his spin? Explain this semi-quantitatively.

c. _____ there are _____, then the _____ that can occur _____ that _____ the _____.

GUIDED NOTES

- d. Example 19: Try to explain the bicycle tire demonstration semi-quantitatively and narratively.
- e. Example 20: A satellite of mass 8×10^3 kg orbits Earth (mass 5.97×10^{24} kg) in an elliptical orbit. (a) In what sense is this a rotating system? (b) The nearest radial distance for the satellite is 6.4×10^7 m, and the farthest radial distance in the elliptical orbit is 6.7×10^7 m. We can take only the satellite into account since the moment of inertia of Earth as an internal object is constant due to the center of mass of the system being essentially at Earth's center. What is the angular speed of the system if the tangential velocity of the satellite during the closest orbital point is 7.5×10^3 m/s? (c) What is the angular momentum at this point in the orbit? (d) What is the angular speed of the satellite at its farthest orbital point? (e) This is an example of Kepler's second law of planetary motion. What do you think this law might state?